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APPLIED MECHANICS REVIEWS

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ON THE ANALYSIS OF COMPLEX ELASTIC STRUCTURES

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I INTRODUCTION

The calculation of complex and highly redundant elastic structures has received a great impetus since the end of World War II. This may be ascribed in the main to two factors; first, to the new techniques developed to meet the ever increasing requirements of greater accuracy and speed in the analysis of aeronautical structures—growing larger and more unusual year by year—and, second, to the revolutionary advent of the electronic digital computer. Naturally, complex structural problems occur also in the mechanical and civil engineering field but the challenge of the pressing problems in aeronautical structures has been probably the most decisive influence in these new developments.

We need not emphasize that the calculation of intricate elastic systems under static and dynamic loads involves, in principle, only simple physical and mathematical ideas. Where the difficulties really arise is in the practical application of the theory on any given problem, which may involve protracted numerical operations necessitating a high degree of organization and which requires also a sure physical insight in setting out the problem. In our search for methods to deal with highly complex structures the combination of the variational principles with continuous analytical functions (e.g. Rayleigh-Ritz method) cannot help us significantly and hence will be ignored in this review.

If, for simplicity and brevity, we restrict our considerations to linearly elastic structures and small displacements we are faced, in fact, in any extensive problem, i.a. with the derivation of a large number of simultaneous linear equations (in the statical and kinematical redundancies) and their solution. In addition, we may have to determine in dynamic problems the eigenfrequencies of a structure. While we stress, in general, the time-consuming task of solving such a system of equations with many unknowns, we do not often realize sufficiently that, as pointed out for example by Langefors (34), the derivation of the coefficients of the equations may itself be an even more formidable undertaking.

To expand upon the last remark let us consider the relaxation or iteration techniques used so successfully in the past to calculate complex elastic systems. These methods originated in the inspired techniques of Čališev (69) and Hardy Cross (70) for rigid jointed frameworks and were developed into a general relaxation philosophy applicable to structural and other physical systems by Southwell (1), (2) and his associates (3), (4). Interestingly enough, the mathematical idea of the relaxation process can be traced back to Gauss, whose passion for numerical work gave birth to many of the novel techniques in relaxation (76). The relaxation procedure when applied to frameworks operates essentially on equations which may also be found by the more standard structural methods. Even here, in such relatively simple problems, considerable numerical labor may be required to determine all the coefficients of the equations. These difficulties are accentuated considerably in intricate continuous structures with reinforcements and cut-outs, e.g., shells, stressed skin wings and fuselages. In such cases the relaxation pattern is usually based on the finite difference approximation to the differential equations of the problem, and long and tedious work may be involved in setting up the basic relaxation operations since these are non-standard. Last but not least, the boundary conditions require considerable attention. Now, with the general availability of electronic digital computers, the interest in the iteration techniques of solution of simultaneous equations is declining and it is probably only for simple stiff-jointed frameworks that such approaches may still have their uses in future.

When we come to more intricate systems and, in particular, to continuous structures, their complexity imposes the use of the electronic digital computer not merely for the solutions of the equations but also for the complete process of structural and dynamic analysis from the input of the initial data to the complete stress distribution, flexibility matrix, eigenfrequencies, etc. This is ideally achieved in conjunction with the matrix methods of structural analysis developed since the war. In the following sections we shall describe the main characteristics of these methods, which are today the most powerful tool in the numerical analysis of engineering structures, and review the main trends in the literature on the subject. No reference will be made, however, to the important questions of numerical accuracy, methods of programming and checking procedures; they, obviously, require a separate review article.

II MATRICES IN STRUCTURAL ANALYSIS

Some of the background leading to a "matrix analysis of structures" follows. To avoid any possible misunderstanding we emphasize that under this heading we do not mean the application of matrix algebra merely to the solution of the linear equations in the unknowns. We would have achieved indeed little, if any, progress if we had confined the introduction of matrix operations to this essentially trivial task. What

we have in mind here is the consistent development in matrix language of the complete structural theory—from the initial data in their simplest possible form to the final stress distribution, eigenfrequencies, etc. Two significant advantages may immediately be quoted for this approach:

First, the use of matrices allows a systematization and simplification of the calculations impossible under any other scheme. Thus, once the initial matrices are assembled, the subsequent operations involve merely elementary matrix algebra. What is more important, however, is that the matrix formulation is the ideal "language" for the electronic digital computer. We can, in fact, program the machine to perform all matrix operations using only the simplest of instructions with respect to location and order of the matrices; this is described in England as a Matrix Interpretive Scheme. But for certain obvious limitations arising from the finite size of the computer this programming allows us to treat a matrix in any instruction practically as if it were a single number and not a rectangular set of numbers. On this basis the program of any individual problem or, indeed, a general purpose program for a large class of structures is easily written. (Hunt (12)).

Second, the application of matrices introduces an exceptional conciseness and transparency of the mathematics which is impossible with the standard longhand notation. In fact, the matrix formulation has proved most fruitful and has presented us with a number of valuable theorems and applications which would have been difficult, if not impossible, to detect by older methods. But to realize fully the implications of these advances we must recast the basic structural principles in their most general form in matrix algebra and condition our minds to think automatically in the matrix language. At the same time nothing more than neat physical thinking and elementary matrix algebra is required. Structural engineers should not let themselves be cowed into believing that for these essentially simple operations it is necessary to know topology and n-dimensional geometry.

Either of the above two advantages of the matrix formulation cannot be realized, in general, on the finite difference approach. Further points showing the superiority of the matrix approach will emerge from the discussion following.

III ON THE IDEALIZATION OF THE STRUCTURE

Before continuing our account of the matrix method let us consider for a moment the complex structure we wish to analyze. If it is a continuous system (but even the simplest frame would serve the same argument), say a wing, fuselage or a civil engineering shell, then its degree of statical or kinematical indeterminacy is strictly infinite. An exact solution of the relevant differential equations—if they can be established—for the given boundary conditions is in most cases impossible, and approximate continuous solutions using the principles of virtual stresses or displacements are, as stated in the introduction, usually not a practical proposition for intricate structures due to the existence of webs, ribs, flanges, stringers, etc. The relaxation procedure often successfully circumvents these difficulties, but for the reasons stated in the Introduction, Section II and later on, it cannot be considered a very efficient procedure for complex systems. To apply, on the other hand, the ideas of Section II we must first perform a process known as the idealization of the structure. This is, naturally, only a more elaborate procedure than the standard assignment of ideal pinjoint properties to the semistiff joints in frameworks.

To start with we cover our structure with a suitable fictitious grid-system of orthogonal or nonorthogonal lines whose intersections we call nodal points. For simplicity let us restrict ourselves, for the moment, to those problems of wing or fuselage analysis where the grid lines are orthogonal or nearly so. A standard idealization of the structure assumes now that we can concentrate all direct stress-carrying capacity of the sheet and certain stringers along grid lines and that the sheet material is then only shear-carrying (Ebner and Köller (72), (73); these references are of great historical interest). The effective flange areas along the grid lines must be estimated (from the yet unknown stress distribution!), but we are not concerned here with such detailed points; see (73) Appendix, and Ref. (5) March 1955. Arising from this idealization the assumption is usually made that the effective flange loads vary linearly between adjoining nodal points and that the shear flow within each field formed by the intersection of pairs of adjoining grid lines is constant. This is the simplest idealization used in aircraft stress analysis when forces or stresses are introduced as unknowns. But it is not our unique choice in the force method. Thus a much more accurate and subtle idealization is the L-matrix technique given in Ref. (5), March 1955; this procedure, to which we shall refer in Section V, may also be applied in the case of plates or shells, where there occurs a rapid stress variation, and may include the bending shear strain effects (Baker (10)). For a structure where the grid lines intersect at a pronounced oblique angle, the force method requires, naturally, another idealization (e.g., Falkenheiner (26)).

When we wish to analyze a structure with the displacement method, using displacements or strains as unknowns, we must obviously think again about the idealization procedure. The majority of the literature on the displacement method (Levy (33)) does actually use the same idealized model as in the force method, but we feel that this is an unnecessarily restrictive step. A much better and more natural approximation is achieved with an idealization based on an assumed linear variation of strains between adjoining nodal points, which dispenses altogether with the idea of a purely shear-carrying panel (see (5) April 1955). This procedure has, moreover, the considerable advantage of being easily applicable to arbitrary directions of grid lines.

It will be apparent from the foregoing discussion that the idealization of the structure and stress or strain pattern—which is inevitable for practical purposes—must depend on the geometry of the structure (e.g. sweptback wing) and the proposed method of analysis (e.g. force or displacement method). But whatever our process of idealization may be we shall ultimately have transformed our given system into an assembly of a large number of components (elements) with an assigned stress (force) or strain pattern between nodal points, e.g. flange-elements with force varying linearly, sheet-elements carrying pure shear, etc. (That the process of idealization involves also geometrical adjustment of the actual variation of cross-sectional areas, thicknesses, etc., need not be emphasized here.) Our problem now is how to analyze this idealized system for any given dynamic or static loads. The latter are usually, though not necessarily, represented by equivalent discrete loads acting at the nodal points.

The fundamental difference of the above approach from the relaxation technique of finite differences as applied in continuous structures will by now be apparent. The former is based on a mechanistic idealization of the structure, the latter, in general, on a mathematical approximation of differential equations.

IV THE PRINCIPLES OF STRUCTURAL ANALYSIS

Any method of *structural* analysis of elastic systems may be reduced ultimately to one of two dual principles, which are most conveniently and generally expressed as the principles of virtual forces and virtual displacements. These principles originate, at least in their more elementary form, in the work of Maxwell, Mohr and Engesser. They are quite independent of elastic laws or the structural material and hence are applicable to cases of nonlinear elasticity and initial (e.g.

thermal) strains. These wider aspects of the two virtual principles are discussed, largely in physical terms, in Ref. (5).

The actual analysis of engineering structures is greatly facilitated, both theoretically and practically, by choosing particularly elegant and concise formulations of the two principles. These are, in our opinion, the Unit Load and Unit Displacement Theorems. Both are much wider in their scope and applicability than the corresponding Castigliano theorems, as usually given. Briefly, the Unit Load Theorem effectively expresses the kinematic relations between strains and displacements by means of statically equivalent stresses, i.e. stresses which satisfy the equilibrium conditions but not necessarily the compatibility conditions. Similarly, the Unit Displacement Theorem effectively expresses the equilibrium relations between stresses and forces by means of kinematically equivalent strains, i.e. strains which satisfy the compatibility conditions but not neccessarily the equilibrium conditions. As with the more general principles, both theorems are valid for nonlinearly elastic structures and may be derived without introducing either the idea of work or strain energy. Once conditioned to these concepts one realizes their power, fruitfulness and conciseness. Both were essentially given, but in a more elementary form, by Maxwell and Mohr* (see literature references in Ref. (5)). The Unit Load Theorem is, of course, the basis of the δ_{ik}-method so much used in Continental Europe. A general discussion of both principles, stressing their duality, may be found in Refs. (5) and (8).

The Unit Load Theorem leads naturally to the force method of structural analysis where stresses (or resultants) are introduced as redundancies, while the Unit Displacement Theorem leads to a structural analysis where displacements (strains) are introduced as redundancies. In what follows, when we state side by side the main characteristics of both approaches, the first statement refers always to the force method (FM) and the second [in brackets] to the displacement method (DM).

After the idealization of the structure, an essential task is the determination of the degree of statical [kinematic] indeterminacy for which simple rules can be given (e.g. (5) and Henderson-Bickley (40)). We then proceed to the selection of static [kinematic] redundancies, which deserves some comments. For continuous structures the degree of indeterminacy is considerably smaller in the FM than in the DM. Moreover, the equations in the unknowns are in such systems better conditioned in the FM than in the DM. But there are other important considerations in the choice of the method, as we shall see in the following section.

Great stress is laid in Ref. (5) on a systematic choice of the force redundancies and on achieving the minimum overlapping of the corresponding stress systems for best conditioning of the equations. It is also emphasized that for symmetry and simplicity of expressions we must not consider redundancies as single forces acting on single physical cuts of the system but as self-equilibrating groups of forces or stresses; this facilitates moreover the analysis and eases considerably the vital task of checking the calculations. These ideas are illustrated in Ref. (5) on wings of admittedly simple form but it is possible and, in fact, indicated to use similar concepts in more complex structures; two of the systems proposed in Ref. (5) derive from the *P*-function given in Ref. (73) in 1947. However, while writing this review the author came across a most remarkable contribution by Michielsen (50) published in 1949, which he had not noticed before, where the identical two systems were proposed for the same reasons. This paper—although not in matrix form—was in many ways far ahead of its time. It includes also an estimation of the effects of plasticity, using effectively the concept of initial strains. It is strange that although the idea of self-equilibrating stress systems for the redundancies goes back into early civil engineering practice (e.g. Kaufmann (71)) aeronautical engineers seem to have been reluctant to adopt these systems although their elegant and effective use was demonstrated by Ebner before the war, (Ebner and Köller (72) and (73)). Even recently some publications on the matrix analysis of structures appear to treat redundancies as single forces.

Although the preceding discussion is centered on the force method it is evident that similar ideas may be applied in the displacement method.

V THE MATRIX FORMULATION OF STRUCTURAL ANALYSIS

Next we outline the matrix theory of structural analysis. Many valuable introductory papers were published on this subject in the early 50's (Falkenheiner (24), Langefors (30), Wehle and Lansing (31)) and we shall discuss the most important contributions in the following section. But it is probably fair to state that the most general and yet simple formulation of the theory was developed in Refs. (5) to (9). This series of papers, in particular (5) and (8), gives a systematic and concise presentation of a matrix analysis with forces or displacements as unknowns and emphasizes, ab initio, the complete duality of both methods. The theory flows naturally from the very simple matrix form of the Unit Load [Unit Displacement] theorems for an engineering system consisting of an assembly of elements.

The matrix formulation of structural analysis by the force [displacement] method starts by setting up four matrices. They are:

(1) a rectangular matrix the elements of which are stresses, statically equivalent [strains, kinematically equivalent] to unit external forces [displacements] in the components of the system when the static [kinematic] redundancies are taken as zero.

(2) a rectangular matrix the elements of which are the stresses [strains] in the components per unit static [kinematic] redundancies and zero external forces.

(3) a diagonal matrix the elements of which are the flexibilities [stiffnesses] of the unassembled components for the prescribed pattern of stress [strain] distribution and may themselves be submatrices.

(4) a column matrix of the prescribed forces [displacements]. In addition, in dynamic problems with a prescribed discrete mass distribution, there is a diagonal mass matrix the elements of which may themselves be submatrices.

Matrices (1) and (2) derive entirely from elementary static [kinematic] considerations and deserve some discussion. Thus, it is important to realize that matrix (1) may be based on *any* suitable stress [strain] system as long as it is statically [kinematically] equivalent to the applied unit loads [displacements]. It is hence to our advantage to select the simplest possible static [kinematic] system; for example, in a wing analyzed by the force method, we may find matrix (1) on the independent spars (Michielsen (49) and Ref. (5)). In much of the published work, on the other hand, where redundancies are taken as single forces at hypothetical physical cuts the much too narrow view is taken that the statically equivalent stress system has to be calculated on the statically determinate basic system remaining after the cuts. Considering now matrix (2) we note that it can be simplified considerably by the systematization and generalization of the redundancies mentioned in Section IV. Many cases will present themselves in practice where we can write down the elements of matrices (1) and (2) by inspection although some simple numerical

*The Unit Displacement Method does not appear to have been given explicitly by these authors, but has been used, even if in a disguised and restricted form, by a number of authors analyzing frameworks by the displacement method; (see, in particular, Ostenfeld, A., Die Deformationsmethode, Berlin, Springer, 1926.

calculations may still be required. However, many structures will occur where this is not feasible or rather not so practical from the programming point of view. Here it may be preferable to derive matrices (1) and (2) by inversion of matrix equilibrium [kinematic] conditions directly on the digital computer; this procedure is obviously greatly simplified by an intelligent selection of the redundancies. In *either* case it is most profitable to use the digital computer not only to calculate the elements of the matrices directly from the initial geometrical data of the structure but also to assemble the matrices themselves. It is occasionally advisable to combine matrices (1) and (2) into one matrix.

The third matrix giving the flexibilities [stiffnesses] of the unassembled idealized components of the structure can be determined most effectively and elegantly—for each type of component and assumed stress [strain] distribution—by the unit load [displacement] theorem. These individual flexibilities [stiffnesses] are, in general, matrices the elements of which are simple functions of the geometry and elasticity of the structure; for simple cases, e.g. flanges with end load varying linearly, standard expressions are available. Since, as mentioned in Section III, the idealization in the force and displacement methods need not be the same, the flexibilities of the components will, in general, not be the inverse of the stiffnesses (Ref. (5), March 1955). Following the discussion of Section II it will be found that for more complex components, e.g. quadrilateral panels, the displacement method yields more readily the stiffnesses, which include, moreover, Poisson's ratio effect. Thus, as stated also in Section IV, the decision to use the force or displacement method, will depend on the geometry of the structure, store of digital computer, etc. For highly complex wings the displacement method will often be found more convenient but, on the other hand, the force method should always prove superior for fuselages.

The above discussion on flexibilities and stiffnesses assumes the standard idealization of the force and displacements mentioned in Section III. When the estimation of the effective flange areas is difficult due to a rapid (and yet unknown) variation of the stress distribution we may apply in the force method a more refined idealization—known as the **L**-matrix technique—which dispenses altogether with any guessing of flange areas. The corresponding flexibility matrix of the flanges, which includes Poisson's ratio effect in the sheet, is, however, no longer diagonal and requires an inversion process (Ref. (5), March 1955).

Whatever approach we may choose, the digital computer must again be used—as for matrices (1) and (2)—to calculate the elements of the flexibilities [stiffnesses] and assemble the matrices.

Using the matrix formulation of the unit load [displacement] theorem to satisfy the kinematic [static] compatibility conditions in the assembled system, we find by elementary matrix operations on the basic matrices (1) to (4), the matrix equation in the unknown static [kinematic] redundancies. Hence, by inversion of the matrix of the coefficients we find the redundancies themselves and ultimately the stress [strain] distribution in the structure.

A further simple matrix operation yields the flexibility [stiffness] matrix of the idealized structure which in combination with the mass matrix yields the eigenfrequencies of the system. Finally, we may derive in conjunction with an aerodynamic matrix the aeroelastic properties of our structure. In most of these applications the elastic properties of the structure enter more directly as the flexibility matrix rather than the stiffness matrix.

The above theory (see also review in Section VI) is extended in Refs. (5) to (8) to include cases of given displacements [forces] initial strains [initial stresses] e.g. thermal or lack of fit effects. The method is also generalized to statically [kinematically] indeterminate basic systems, which is, of course, effectively equivalent to an inversion of the matrix of the coefficients by partitioning. Finally, we note that in the foregoing methods, contrary to the relaxation technique, the satisfaction of any given boundary conditions is straightforward if not automatic.

Programming the method

We have discussed the vital use of the digital computer to assemble the matrices (1) to (3). It is now extremely simple, on the basis of the matrix interpretative scheme (see Section II), to write general purpose programs—or for that matter any special program for any particular problem—for the force [displacement] method (e.g. Hunt (12)). Thus, starting from the simple initial geometric and elastic data and the prescribed loads [displacements] we can calculate on the digital computer *in one continuous operation* the complete stress [deformation] pattern and obtain also the flexibility [stiffness] matrix. A further simple subroutine yields readily the eigenfrequencies of the system. We emphasize that no special analysis (e.g. semi-rigid assumption, etc.) is necessary or called for in dynamic applications. Conciseness and unity are the operative words in the matrix assembly methods of structural analysis.

The method of cut-outs and modifications

We referred previously to the great advantage derived, in application of the force method, by a systematic and uniform pattern of the static redundancies. This is possible even in very complex systems—if there are no cut-outs in continuous structures or missing members disturbing the symmetry in frameworks. Although the general matrix methods may naturally be applied to any such irregular system they are inevitably more complicated in the presence of cut-outs, since special redundant systems have to be selected (Ref. (5), May 1955). This disturbs markedly the uniform pattern of the matrices (1) and (2) and increases the labor of checking them.

To circumvent these considerable practical disadvantages, a new technique was suggested in Ref. (5), March 1955. Assume that we fill in all cut-outs or missing members and obtain a continuous structure easily and smoothly analyzed by the standard programs for the given external loads (or thermal effects). We impose next on the fictitious new elements such initial strains that the total stresses in these elements due to both loads and initial strains are zero. These components are then effectively nonexistent and we regain our original structure (with cut-outs) the stresses in which under the given load system are identical to those in the continuous system under loads and initial strains. Now the magnitude of the initial strains and, what is more important, the corresponding stresses can be expressed very simply (in closed and explicit matrix form) in terms of the known stresses in the continuous structure due to external loads alone. Thus, we obtain finally the true stress distribution in the structure with cut-outs solely in terms of the stresses in the continuous structure under the same loads. The technique is surprisingly simple, easily programmed, and has proved very effective in practice. Goodey (41) gave later an independent derivation of the same device based on a variational argument. We stress that the technique is, in principle, different from the perturbation methods put forward, for example, for circular fuselages with cut-outs, in which a *separate* stress analysis is required for the perturbation forces.

The above method has been generalized in Refs. (6) to (8) to derive the corrections to the stress [strain] distribution and flexibility [stiffness] matrix of a structure due to modifications to some structural components. Here again we find by a similar only slightly more elaborate technique, both in the force and displacement methods, the stress [strain] distribution in a modified structure simply in terms of the stress [strain] distribution of the original structure under the same

loads or thermal effects. Interestingly enough the author discovered lately that Michielsen and Dijk (51) gave as early as 1953 a procedure which, while not leading to closed expressions and a matrix presentation, contains much of the idea as far as the force method is concerned. Thus they operate with perturbation forces but, contrary to other authors, are able to determine easily the corrective stresses from a pre-existing general solution of the redundant structure (49).

It will be apparent from the preceding discussion that with the matrix methods we have an effective and practical tool of structural analysis to determine the *over-all* stress distribution in intricate engineering structures. However, there are cases when we may have to refine the stress and deformation pattern at marked discontinuities of structure and loading. For example, in the process of diffusion of a load we may wish to obtain a more detailed picture of the shear flow distribution along the flange. Here continuous analytical techniques used locally will still prove probably the most suitable.

In our description of matrix methods we have not considered the combination of differential equations and matrix algebra proposed so elegantly by Jenkins (52) and others for shells. This approach is no doubt most useful for smooth shells of relatively simple geometry but does not fit into our requirements for a standard method to analyze intricate structures.

VI SURVEY OF IMPORTANT PAPERS ON MATRIX ANALYSIS

In this section we give a historical survey of the more important papers on matrix analysis not discussed in the previous section. We exclude at present publications which embrace a more philosopho-mathematical terminology and presentation; they will be considered separately in Section VII.

Following a number of contributions introducing matrices in special problems, to which we cannot do justice here (e.g. Jensen and Dill (14), Kempner (15), Benscoter (17), (18), Plunkett (24)), Falkenheiner (25), (27) was the first to develop the basic principles of the matrix force method, with particular reference to standard orthogonal stringer-sheet assemblies under loads. His account includes a clear statement on the importance of self-equilibrating load-systems as redundancies, and also the derivation of the flexibility matrix for the assembled structure. The equations in the unknowns were obtained from the principle of least work. The systematization of the operations and the physical background were inevitably not yet very advanced and the derivation of the flexibilities of the elements did not appear as an integral part of the same basic theory (see Section V). Falkenheiner's original contribution was, nevertheless, more advanced than much of the subsequent work on the force method. In (26) the same author extended the method to statically redundant basic systems and showed how to take approximate account of the direct stress-carrying ability of swept panels. Subsequently, Langefors (31) gave another independent account of the force method using once more the minimum strain energy approach; he referred also to the electrical network analogues of Kron (see also (29)). Wehle and Lansing described clearly in (32) the basic force method and the flexibility matrix of an assembled structure. The discussion of redundancies was rather vague with no mention of self-equilibrating load systems. They tabulated the flexibilities of various components and suggested to derive the matrices (1) and (2) of Section V by writing down and solving formally all the equilibrium equations with certain of the internal forces taken as redundancies. Prior to these last two papers, Lang and Bisplinghoff (30) obtained a matrix formulation of a strain energy analysis for a sweptback wing given previously by Levy (19). Langefors in (34) discussed very briefly the relative merits of the displacement and force methods but considered only the latter in detail. He used, interestingly enough, the matrix formulation of the unit load theorem (referred to in Section V) but in a more restricted form, and considered it as following from strain energy or Kron's work (see Section VII). The paper proposed also to solve highly redundant systems by splitting them into a number of subsystems, each of which is analyzed separately; see also (31) where the method was given in a more restricted form. The procedure is, of course, effectively the method of a statically redundant basic system. A recent modern exposition of the elements of the force method may be found in Denke (38) who effectively derived the matrix equation in the unknowns from a matrix formulation of the unit load theorem (again expressed in a more restricted form) which he quotes as arising from the Maxwell-Mohr approach. He argued correctly that, since the strains in the Maxwell-Mohr relations may be due to any cause, the incorporation of thermal strains or other initial strains is straightforward. Denke selected as redundancies forces at hypothetical cuts, and determined matrices (1) and (2) by inversion of the equilibrium conditions. In a further publication Denke (40) showed how the procedure could be applied to certain nonlinear cases. Finally, Goodey (44) in a recent series of papers investigated exhaustively, by a combination of matrix and analytical methods, circular fuselages, and applied successfully the cut-out technique of (5) and (41).

The literature on the displacement method is not so extensive and none of the papers quoted below developed the matrix assembly technique. Livesley in (37) presented a special analysis of stiff-jointed frameworks using the concept of stiffness. The first semi-matrix scheme for aeronautical structures was proposed by Levy (33). He considered as components the spars and ribs for bending and the cells for torsion, the stiffnesses of which were obtained by inversion of the flexibilities. Thoman generalized this procedure in (43). In two reports (45) and (46) Williams proposed the application of a displacement method to wing stress and oscillation problems. The method is not, however, a matrix procedure in our sense. Thus, this author approximates the partial differential equations problem by finite difference equations the coefficients of which form the stiffness matrix, the inversion of the latter gives the flexibility matrix from which the displacements, strains, stresses and eigenmodes are found. Matrices enter only in the inversion of the stiffness matrix.

A mixed force/displacement method was proposed in (48) and (49) by Klein who writes down the equilibrium equations and the force displacement relations for all elements. These two groups of equations are solved jointly for the forces and displacements. The matrix formulation appears only in the final equations and the number of unknowns is inevitably very high.

Not unnaturally the effect of modifications to components has occupied the attention of a number of authors (we mentioned Michielsen in Section V). McNeal and Kosko considered the rather restricted problem of finding the change in the flexibility matrix if a block of coefficients of the stiffness matrix of the structure is changed. Strictly, however, this does not solve the problem of modifications since we have still to determine i.a. the influence of the modifications to the structural components on the stiffness matrix, which for complex structures requires the general matrix solution described in Section V.

Examples of specific applications of matrix methods to oscillations of elastic systems may be found in Fraser, Duncan, Collar (75), Targoff (20), Thomson (23) and Ehrich (47). References (20) and (23) are of particular interest since they form the basis of a technique developed further by some German authors (see Section VIII). Thus Targoff investigates the eigenfrequencies in bending and torsion of beams with a

stepwise variation of cross section and discrete mass distribution. He relates the characteristic quantities (forces, deformations, etc.), written as a column matrix, at the two ends of each uniform element by a transformation matrix which follows from equilibrium and stiffness considerations. A further transfer matrix connects the characteristic quantities on either side of a mass. By chain matrix multiplication Targoff obtains a matrix equation relating the characteristic quantities at the two extreme ends of the beam. This is done both for bending, torsion, and coupled bending-torsion modes; reference is made to some previous work of Pipes on torsion. The frequencies are ultimately determined by application of the Holzer-Myklestad iteration technique. Thomson (23) presented very clearly the same technique for bending vibrations alone, but included the case of continuously varying cross sections and uniform mass distribution. In the latter problem the relation between the column matrices at the two ends of each element are found from the differential equation in the deflection.

VII WORK OF MORE GENERAL INTEREST

It is evident that the general principles of matrix analysis of assembled structures may also be applied to a large class of physical problems concerned with assemblies of elements and governed by linear equations. All that we need is the "translation" of the physical quantities occurring in the structural problem into the corresponding quantities of the new problem. Interestingly enough, the first major applications of matrix algebra occurred in electrical network analysis. We refer here to the elegant work of Pipes (53). The same author introduced matrix operations in heat transfer (54) and other problems (13). A large literature has grown in the meantime on the matrix analysis of the electrical networks, etc., which does not concern us here.

The obvious similarities between all these linear problems has led some authors to consider the question of an assembly from a more general point of view. Kron has pursued this subject since 1939 in an unusual series of papers starting from electrical network analysis and using them as analogues for other problems. It must be pointed out, however, that the structural contributions (57) and (59), even if stimulating, are of a much more restricted character than the theory expounded in Section V. The method of tearing proposed by Kron is, in fact, directly related to the method of a statically [kinematically] redundant basic system referred to previously. Thus, this author divides his complex structure into a number of subsystems which may be analyzed separately (components!). These subsystems are then joined together by links which in our terminology are the static (or for that matter kinematic) redundancies of the assembly of the subsystems; the connection tensors put forward by Kron for these links are effectively our matrices (1) and (2). It is possibly difficult to do full justice to Kron's presentation of matrix analysis due to the lack of worked out examples.

A still more ambitious undertaking of a matrix analysis of an assembly of elements in linear physical problems was given very recently by Langefors (60), based on the purely mathematical approach of linear algebra. He considers two dual techniques which in the structural field are the force and displacement methods. The properties of the (idealized) components are assumed somehow fixed (by the structural engineer presumably!) and are taken to be identical in both the force and displacement methods. This imposes, as discussed in Section V, an unnatural restriction on the displacement method in stringer sheet assemblies. No direct procedure is given for obtaining the flexibilities and stiffnesses of unusual elements except by referring to strain energy or other methods. It follows that the more sophisticated **L**-matrix idealization could not be devised by this approach. In structural terms, Langefors develops first the displacement method and uses this to establish the force method. The effect of initial strains is included—indeed the concept of statical redundancy arises from a consideration of the possible stress systems generated by initial strains—but it is not indicated, for instance, how to deal with the practically important case of linearly varying temperature in a flange. The unit load [displacement] theorem is presented in matrix form as part of the so-called co-transference relations, but the quoted expressions are again more limited than in Ref. (5).

Langefors' effort, although inevitably rather limited in the structural field, deserves great praise for its rigorous mathematical presentation, which should appeal to applied mathematicians less interested in a geometrico-physical approach. The structural scientist and engineer should find, in general, the more general derivation of a matrix analysis stemming from the unit load and displacement theorems based on the physical ideas of statical and kinematical equivalence more illuminating and fruitful.

VIII SOME RECENT GERMAN CONTRIBUTIONS

We mentioned in Section VI that the techniques of Targoff (20) and Thomson (23) were applied and developed considerably by a number of German authors investigating the statics, dynamics, and stability of stiff-jointed frameworks, beams, and rods. Two slightly different approaches may be distinguished in these contributions. Pestel (61) considers oscillations of systems with discrete masses and uses for each uniform element a generalized form of the transformation matrix of Targoff-Thomson which he derives by a similar argument. At supports, branches, masses or other discontinuities, however, he augments the transformation matrix by introducing new unknowns, e.g. inertia forces, for which he sets up additional equations. Falk (63) in a similar analysis of a static problem uses at the supports, etc., a generalized form of Targoff's transfer matrix. As in the American papers, Pestel and Falk obtain ultimately by chain matrix multiplication the matrix relation between the column matrices of the characteristic quantities at the ends of the system. For static problems in two (three) dimensions there follow three (six) equations in the unknowns, while for dynamic problems the secular equation is obtained from the homogeneous equations.

A slight variation of the above technique is given by Fuhrke (65), (66), Schnell (64), Falk (62) and Marguerre (56) who analyze both instability and oscillation problems with discrete and continuous mass distribution, the latter application being a development of the corresponding procedure by Thomson. These authors use the standard differential equations for the deflection in each uniform element to determine once more a matrix relation between the characteristic quantities at the two ends of the element. Using further transfer matrices at supports they finally derive, similarly to Pestel (61), the transcendental or polynomial equation in the eigenvalues. Marguerre presents in (56) a very clear account of the method and generalizes it considerably.

This group of German papers is no doubt useful in the restricted field of beam assemblies and frameworks in two or three dimensions. In particular, the analytical derivation of the transcendental secular equation for buckling and oscillation problems will be found neat and helpful. However, for the numerical analysis of static and dynamic problems of such engineering systems the general matrix methods of Ref. 5 and Section VI should prove, in general, superior. Any refinements of the classical beam theory, like shear strain and torsion-bending effects, are easily included in the latter techniques. Also the programming on the digital computer, assuming discrete mass distribution, is standard and simpler.

The author wishes to thank his collaborators Messrs. Kelsey, Lianis and Valanis for the great help given in reading and reviewing much of the literature quoted in the bibliography.

The author regrets that due to a lack of space he did not find it possible to comment on many valuable NACA and MIT reports on applications of matrix algebra to static and dynamic problems.

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(See also Revs. 2425, 2433, 2438, 2450, 2456, 2475, 2504, 2511, 2526, 2606, 2655, 2664, 2673, 2696, 2698, 2700, 2711, 2712, 2726, 2734, 2791, 2792, 2793, 2807, 2832, 2838, 2847, 2908)

Book—2398. Brand, L., Vector analysis, New York, John Wiley & Sons, Inc., 1957, xiii + 282 pp. $6.

Book contains a broad and easy introduction to vector analysis. The mathematical correctness is nevertheless not given up. Vector and tensor analysis are now widely used in theoretical physics, analytical mechanics, and in other parts of physics. The textbooks usually include a chapter about the vector and tensor analysis necessary for the book in question. For the sake of clearness many students want to read the whole formal theory in one single book. For them this elementary exposition can be recommended. The book can be looked upon as a shortened and simplified edition of the same author's "Vector and tensor analysis."

After a full exposition of vector algebra, chapters follow on line vectors, differential invariants, and integral theorems. Forty pages are devoted to applications to dynamics and fluid mechanics and as many to electrodynamics. The last 26 pages contain vector spaces in more than three dimensions.

The value of the book is greatly increased by about a hundred exercises. Some of these are fully solved, and answers are given to the others.

H. Faxen, Sweden

2399. Baez, A. V., Model to illustrate some properties of vectors in three dimensions, *Amer. J. Phys.* **25**, 6, 380–381, Sept. 1957.

Vectors represented by thin rubber tubing may be used to illustrate the polygon rule for vector addition in three dimensions. The elastic model lends itself to illustrate multiplication by a scalar, the difference between vectors, a displacement vector which varies with time and the fact that the time derivative of a displacement vector is tangent to the trajectory even when it is not a plane curve.

From author's summary

Book—2400. McConnell, A. J., Application of tensor analysis, New York, Dover Publications, Inc., 1957, xii + 318 pp. $1.85. (Paperbound)

This new Dover edition is an unabridged and unaltered reproduction of the work originally published under the title "Applications of the absolute differential calculus".

The editors

2401. Argyris, J. H., A matrix theory of statics (in German), *Ing.-Arch.* **25**, 3, 174–192, 1957.

2402. Argyris, J. H., and Kelsey, S., Structural analysis by the matrix force method with application to aircraft wings, *Jahrbuch Wissenschaft. Gesellsch. Luftfahrt* 1956, 78–98.

The present papers essentially give the content of the authors earlier papers [AMR **10** (1957), Revs. 54, 55] in a more concise form, with the addition of some important generalizations. The first paper is an excellent introduction into the consequent use of matrix methods in statics of frameworks and particularly points out the duality between the unit-load and the unit-displacement methods; the question of how to deal with cut-outs is answered in a very general form. The second paper goes into details of wing structures treated by the unit-load method, again with special reference to the problem of cut-outs and of post-modifications of a calculated system; it is found possible to express the new stresses solely in terms of stresses of the original system prior to modifications; an example calculated on a digital computer (for which the matrix method is particularly effective) illustrates the advantages of the new procedure

K. Marguerre, Germany

Book—2403. Buck, R. C., Advanced calculus, York, Pa., The Maple Press (*Inter. Series in pure and appl. Math.*), 1956, viii + 423 pp. $8.50.

This book on advanced calculus is intended for a two-semester course for students of mathematics, physics, and engineering at the junior, senior and first-year graduate level.

Its seven chapters cover the following items: Elementary topology, Functions, Integration, Convergence, Differentiation, Applications to geometry and analysis, Elements of differential geometry. Every chapter contains a well-chosen number of exercises. An appendix on Foundations of the Numbers System completes the volume.

In reviewer's opinion this book, which presents a systematic and modern approach to the differential and integral calculus of functions and transformations, will be especially welcomed by graduate students and research workers who need to apply mathematics in their work.

E. Volterra, USA

2404. Shulgin, M. F., The reduced kinetic potential of a system of differential equations (in Russian), *Trudi Sredneaz. un-ta* no. 37, 59–63, 1954; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7160.

An expansion of the Liouville type is applied to a system of differential equations of normal form and an auxiliary system of the Lagrange type constructed of twice the order of the original system. I. S. Arzhanykh

Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

2405. Shekis, A. M., A method of integrating the differential equation of the constrained oscillations of a system with the degree of freedom energized by an arbitrary perturbing force (in Russian), Problems of dynamics and dynamic strength no. 1, Riga, Akad. Nauk Latv. SSR, 1953, 165–183; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8049.

The equation is examined of the constrained vibrations of a system with one degree of freedom:

$$\ddot{y} + k^2 y = F(t)$$

in which $F(t)$ is an intermittent, perturbing force. The solution is divided into time intervals in each of which $F(t)$ is continuous. For the nth interval, the equation is sought in the form

$$y_n = C_n \sin kt + D_n \cos kt + \frac{1}{k^2} F_n(t) - \frac{1}{k^4} \ddot{F}_n(t) + \frac{1}{k^6} \ddddot{F}_n(t)$$

(reference being made to N. I. Bezukhov), C_n and D_n being determined by the initial conditions, the lengths of all the sections and the values of F and its derivatives on the boundaries of all the sections.

Solutions are analyzed in detail for different concrete forms of a periodically-discontinuous, perturbing force $F(t)$.

E. P. Popov

Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

2406. Churchill, S. W., Approximate operational calculus in chemical engineering, *AIChE J.* **3**, 2, 289–293, June 1957.

Approximations are suggested to extend the usefulness of operational calculus in solving boundary-value problems of interest to the chemical engineer. General approximations are outlined and specific ones illustrated. The use of computing machines with operational calculus is also considered.

From author's summary

2407. Vodicka, V., Elementary cases of the Dirichlet problem for elliptical domain on a plane (in German), *ZAMP* **8**, 4, 309–313 (Short Notes), July 1957.

The Dirichlet problem is solved for an elliptic domain if the boundary values are given by a polynominal defined in the interior of the ellipse. The solution is a finite expansion in harmonic polynominals. From author's summary

2408. Saroja, Mrs. B. V., An investigation of the solution of the torsion problem for a pierced triangular shaft by relaxation methods, *J. aero. Soc. India* **9**, 3, 35–43, Aug. 1957.

Relaxation solutions are obtained for the Saint-Venant torsion of an equilateral triangular shaft with three symmetrically placed circular holes. A seven-point finite difference approximation of Poisson's equation is used. The nets used are triangular and are adjusted as one-sided differences to fit best the external and internal boundaries.

Evaluations are made for three different mesh spacings. Comparison is made with previous solutions which indicated better liquidation of residuals and increased accuracy.

M. V. Barton, USA

2409. Wrench, J. W., Jr., The calculation of propeller induction factors, *David W. Taylor Mod. Basin Rep.* 1116, 17 pp., Feb. 1957.

An interesting mathematical discussion aimed at improving some standard results in propeller theory by use of asymptotic formulas more accurate than abridged Nicholson formulas for modified Bessel functions of first and second kind.

New results are based on work of Lehmer and are used to estimate errors produced by use of Nicholson's formulas.

Final comparisons indicate Nicholson formulas to be accurate to within 5×10^{-3}. Improved results of author will permit increase in accuracy by at least one order of magnitude, but this increased accuracy is scarcely important in application of propeller theory.

B. W. Augenstein, USA

Book—2410. Tricomi, F. G., Integral equations; New York, Interscience Publishers (Pure and applied mathematics, Vol. V), 1957, viii + 238 pp. $7.

The importance of integral equations to the engineer and physicist has been reflected in the recent outpouring of books in this important mathematical field. Unfortunately, many of these have been unsuited to the applied scientist because of the abstract way in which they have been written and the rather extensive mathematical background necessary to understand them. The author, well known in the field of differential and integral equations, has sought to minimize the first defect by giving concrete illustrations to motivate the abstract concepts. The second defect is handled by inserting in the body of the text some of the mathematics necessary for an understanding of integral equations proper. It is regrettable that the author felt that the present text of only two hundred and twenty pages would have become too large if more of the applications of integral equations were included. Reviewer feels that the excellence of the book would have been further enhanced if applications from the fields of elasticity and vibration theory were included.

As to the contents, the book includes a discussion of the Volterra integral equation, Fredholm equations, and the Hilbert-Schmidt theory. A final chapter deals with some singular and nonlinear integral equations. Thirty-two exercises are given at the end of the book.

Carefully and clearly written, this text is highly recommended to anyone desiring to acquaint himself with this important branch of mathematics. E. J. Scott, England

2411. Istomin, N. V., Tensor of moments for a system of connected vectors and its application in mechanics (in Russian), *Prikl. Mat. Mekh.* **20**, 3, 434–438, May–June 1956.

This paper relates to statics; more specifically, it investigates the properties of the Hamiltonian center (H.C., for short). It introduces a definition in connection with a system of n connected vectors in the three -space in the form of an affine orthogonal tensor of the second rank of the form $S_i = x_j^i F^j$, where F^j are given vectors and x_j^i are the abcissas of their points $A_i(x_i^1, x_i^2, x_i^3)$ of application. The properties of the H.C. are investigated in terms of this concept, which leads to the following theorems; (1) the position of the H.C. is invariant if all vectors are rotated by an angle φ about the axes passing through A_i and perpendicular to the central plane subject to some additional geometrical conditions; (2) the position of the H.C. is also invariant if all vectors are rotated by an angle φ about an axis of an arbitrary direction; the necessary and sufficient condition for this is that the principal vector and the principal moment of the system be collinear; (3) a rotation of all vectors (with the principal vector different from zero) about axes parallel to a given direction and passing through A_i results in a motion of the H.C. on an elliptic arc.

N. Minorsky, France

Book—2412. Dienes, P., The Taylor series, an introduction to the theory of functions of a complex variable, New York, Dover Publications, Inc., 1957, xii + 555 pp. $2.75. (Paperbound)

This new Dover edition is an unabridged and unaltered republication of the first edition with errata incorporated into the text.

It is published through special arrangements with Oxford University Press. The editors

2413. Mertvetsov, M. A., A method of approximate solution of nonlinear functional equations (in Russian), *Izv. Kazansk. fil. Akad. Nauk SSSR, Ser. fiz.-mat. i tekhn. Nauk* no. **8**, 154–163, 1955; *Ref. Zb. Mekb.* no. 11, 1956, Rev. 7625.

In some problems relating to the control of the oil-bearing contour, a system of nonlinear, algebraic equations is obtained. It is suggested to solve such equations by a general method, founded on the iterative expression

$$x_{n+1} = x_n - [1 + \tfrac{1}{2}\Gamma_n P''(x_n)\,\Gamma_n P(x_n)]\Gamma_n P(x_n)$$

in which

$$\Gamma_n = [P(x_n)]^{-1}$$

and $y = P(x)$ is a continuous operation, triply differentiated according to Fraichet (?), and determining the correspondence between the two standardized Banach (?) spaces. In this regard, by making certain supplementary assumptions, proof is given of the theorem of reality and singularity. An example of the application of the general theorem is given.

V. P. Pilatovskii
Courtesy Referativnyi Zburnal, USSR
Translation, courtesy Ministry of Supply, England

2414. Pyatin, Yu. M., An operator method for the algebraic solution of equations describing transitional established processes in linear systems (in Russian), Elements of calculation for precision instruments, Moscow, Oborongiz, 1954, 154-164; *Ref. Zb. Mekb.* no. 12, 1956, Rev. 8050.

The question is examined of improving the efficacy of determination of unknown systems of linear algebraic equations corresponding to a linear electromechanical linkage. It is found, for the case analyzed, that auxiliary relationships exist between the determinants of the system, facilitating calculation by Kramer's formula. An example is given of the calculation of an electrical, star-delta connection. I. I. Eterman
Courtesy Referativnyi Zburnal, USSR
Translation, courtesy Ministry of Supply, England

2415. White, H., and Christie, L. S., Queuing with preemptive priorities or with breakdown, *Operat. Res.* **6**, 1, 79–95, Jan./Feb. 1958.

Analysis of queuing systems with pre-emptive priorities, i.e., newly arrived items of highest priority go directly into "service", the item in service returns to head of queue of its own priority class. Service facility breakdown is shown to be equivalent to arrivals of items with pre-emptive priority. The results are compared with those of Cobham [title source **2**, p. 70, 1954], Halley [title source **2**, p. 341, 1954], and Dressen and Reich [P846, The Rand Corp, 1956], who evaluated results for priority system in which highest priority items move to head of the line. The formulations are restricted to single server queue systems, as was the work of J. W. Cohen [Phillips *Communications News* **16**, p. 105, 1956] which is not included in the authors' references. Poisson arrivals and exponential service are assumed for the highest priority items; the effects of pre-emptive priority on the service time distribution of lower priority items is discussed in detail. Under a number of reasonable assumptions these latter distributions remain exponential (although the mean value may change, as in case when the pre-empted item must repeat all phases of service). The highest priority class is not affected by the existence of lower classes for any service time distribution. While results for mean line length and mean waiting time for highest priority items are obtained with little difficulty, the same parameters for the lower priority items require considerable calculation.

Modified equations are given for the case of breakdowns assuming that "breakdowns" (which are the highest priority items) do not arrive during the servicing of a breakdown. Pre-emptive priority is probably most relevant (1) for systems like communications networks that handle a large volume of routine items and a few very important items and (2) for systems in which the effect of breakdowns is important. The pre-emptive discipline is advantageous mathematically for the ease of a Markov description of changes of state. The disadvantage lies in the difficulty of defining service time distributions for lower priority items. While well written, the paper would be easier to follow if the sections were numbered and the equation enumerations were extended beyond the first half of the paper.

E. Koenigsberg, USA

2416. Meisling, T., Discrete-time queuing theory, *Operat. Res.* **6**, 1, 96–105, Jan./Feb. 1958.

Analysis of single-sewer queuing system in which time is treated as a discrete variable, i.e., customers arrive only at given time marks and service times are discrete multiples of Δt. Arrivals obey a binomial probability distribution. The results in the limits $\Delta t \to 0$ are in agreement with the continuous time case. Transition probabilities are related to the probability of arrivals during a service time (similar to queuing with constant holding times). The mean queue length is determined by use of generating functions, following the example of Crommelin [*P. O. Elec. Eng. J.* **25**, 1932] for queuing with constant holding times.

A special case when service time distribution is a geometric progression is included; the results are identical to the continuous time case with Poisson input and exponential service time. The author suggests that the discrete time approach may allow simpler solutions of continuous time problems by considering the discrete time case and performing a limiting process.

Author is to be commended for the clarity and conciseness of his presentation. E. Koenigsberg, USA

Book—2417. Lindsay, R. B., and Margenau, H., Foundations of physics, New York, Dover Publications, 1957, xi + 542 pp. $2.45. (Paperbound)

This is a new Dover edition (in essentially unchanged form) of a well-known book first published in 1936. The original aim of the book was to bridge the gap between the popular presentations of the methodology of physics and the rigorous treatises of theoretical physics. This aim has been admirably achieved. The clarity of exposition makes the book most delightful reading.

The book is strongly recommended to engineers and engineering scientists who are concerned with the logical basis of physical theories. It is obvious that a student of nuclear engineering or space technology would need such a basic understanding. But as physics and mathematics are of growing importance to all fields of engineering, a book like Lindsay and Margenau's, which was written as interestingly as a novel, should be recommended to all engineers.

It is unfortunate that physics as taught in most of our schools and colleges today does not emphasize the basic logical aspects. Often we find a text puts forward a hasty statement of a physical law, and then proceeds at once to all kinds of applications, without returning to consider the logical difficulties and subtleties of the ideas involved. Or, specialized as most engineering courses are, questions of the more subtle nature are often postponed to another course which the student will have little chance of taking. The present book fills this gap nicely.

The chapter titles are, in order: The meaning of a physical theory, space and time in physics, the foundations of mechanics, probability and some of its applications, the statistical point of view, the physics of continua, the electron theory and special relativity, the general theory of relativity, quantum mechanics, the

problem of causality. A list of selected readings, added for the new edition, ends the book.

Y. C. Fung, USA

Mechanics (Dynamics, Statics, Kinematics)

(See also Revs. 2417, 2444, 2468, 2527, 2565, 2592, 2838, 2842, 2843, 2844, 2845)

2418. Piszczek, K., Influence of geometrical and dynamical constraints on resonance regions in the problem of dynamical stability of a thin-walled bar with open cross section (in Polish), *Rozprawy Inz.* **5,** 2, 207–225, 1957.

The boundary curves of the resonance regions are determined for a thin-walled bar with open cross section, loaded by a longitudinal force $P = P_0 + P_1 \cos \omega t$, with the following assumptions: (1) The bar is supported at both ends in hinges, with the possibility of deplanation of the cross section; (2) the cross section has one axis of symmetry; (3) the force is attached at an arbitrary point of the axis of symmetry. The system of partial differential equations of the problem is derived from W. Z. Wlasow's static stability equations by completing the latter with dynamical terms. The method of separation of variables gives an ordinary system of differential equations, which in the vector-matrix notation has the following form

$$C_1 T + (A_1 - B_1 P) r T = 0$$

where the symbols T, A_1, B_1 and C_1 denote matrices.

On the basis of W. W. Bolotin's papers on dynamical stability problems, which, as is shown, can also be used in the present problem, the determinant criteria for the boundaries of the resonance regions are derived.

Taking only one term of the determinant into consideration, approximate boundary curves of the basic resonance region are derived.

Some particular cases of boundary curves are discussed: (1) The cross section has two axes of symmetry and the force is attached to the centroid of the cross section or to an arbitrary point of the symmetry axis. (2) The cross section has one axis of symmetry and the force acts at the center of transversal forces.

The influence on the magnitude of the resonance regions of the position of the force (dynamical constraints) and that of some cross section parameters (geometrical constraints) are discussed on a numerical example.

M. Piatek, Poland

2419. Godzevick, I. N., The fundamentals of dynamic analysis. Summary of lectures for Civil Engineering Students. Part I—Oscillations (in Russian), Sverdlovsk, Uralskiy politekhn. in-ta 1955, 80 pp. + illus.; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7795.

2420. Veselovsky, I. N., A simple method of determining the deflection of falling bodies due to terrestrial rotation and the theory of the Foucault pendulum (in Russian), Mekhanika, (MVTU **50**), Moscow, Oborongiz, 1956, 120–123; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8025.

For approximately calculating the Coriolis inertia force it is assumed that the relative velocity of the falling body is vertical. The case of vertical fall on the equator is first considered. The general case of falling on a latitude ϕ is referred to the case of falling on the equator by substituting $\omega \cos \phi$ for the angular velocity of rotation ω of the earth. The equations thus obtained coincide with the first approximation obtained by the usual method of expansion according to powers of ω.

The elementary theory of the Foucault pendulum is analyzed similarly.

G. N. Sveshnikov

Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

2421. Litvin-Sedoy, M. Z., The equations of motion of a body, each part of which has a single degree of relative freedom of motion (in Russian), *Uch. zap. Mosk. un-ta* no. 172, *Mechanics* **5,** 177–190, 1954; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8017.

The equations of motion are derived for a system of $s + 1$ bodies (G_0, G_1, ... , G_s) on the following assumptions: (1) The system has applied linkages enabling rotation without slipping of each of the bodies G_1, G_2, ... G_s about one of the axes, immobile in the body G_0; (2) The angular displacements of the bodies G_1, G_2, ... G_s, relatively to the body G_0, are not kinematically interrelated; (3) the masses of the bodies of the system are not interrelated; (4) the relative rotations of the body G_i ($i = 1, \ldots . s$) are related to the internal forces in the system.

With an arbitrary distribution of the masses of the bodies in the system and similar arrangement of the axes of rotation in the fundamental body, the general equations of motion of the said body are derived. Particular cases are analyzed.

V. A. Sarychev

Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy of Ministry of Supply, England

2422. Raven, F. H., Velocity and acceleration analysis of plane and space mechanisms by means of independent-position equations, *J. appl. Mech.* **25,** 1, 1–6, Mar. 1958.

Author applies the complex exponential representation of vectors to the analysis of mechanisms. The equation which expresses in this form the position of a point taken on a member of a mechanism, as a function of the geometry of the mechanism, is called a position equation. For the determination of the velocity and the acceleration of a given point taken on a member of a mechanism, author goes out from a fixed point (called original ground point) of the mechanism. Following the encountered members (fixed members included), he traces all the independent ways connecting the original ground point to the considered point. Two routes are called independent when at least one member is not common to the two ways. Author notes that, except for complex mechanisms, there are only two independent routes.

For each independent route, author writes the position equation (we suppose there are only two independent-position equations); then he equates these two equations and obtains a complex vector relation (1) containing the complex expression of the vector joining the considered point to the original ground point in function of the complex expression of the different members. The velocity of the considered point is obtained by differentiating (2) the relation (1) with respect to time. Equation (2) is then separated in its real and imaginary parts and gives the real elements to be substituted in the vectorial complex expression of the velocity of the point. The acceleration is obtained by a new differentiation. The procedure is applied to a crank-shaper mechanism, to direct contact mechanisms, and to a complex mechanism. The possibility of applying the method of independent-position equations to space mechanisms is shown and the case of a space linkage is treated.

D. DeMeulemeester, Belgium

2423. Rosenauer, N., Synthesis of three-bar mechanisms when angular velocities, angular accelerations, lengths and positions of some links are imposed (in German), *Maschinenbau-Tech.* **6,** 329–333, June 1957.

Referring to his earlier paper in which complex numbers were applied to the synthesis of a three-bar mechanism for which the angular velocity limits of the driven oscillating link were given, author uses the same basic equations for the resolution of the title problem. The general procedure is recalled: the vectorial equation of the polygon formed by the four bars a, b, c, and d is written in the complex exponential form; this equation is twice differentiated with respect to time, giving a first equation for the velocities and a second one for the accelerations.

These three equations are resolved in order to obtain three of the four links in proportional complex expression. These three basic equations are used to realize the synthesis of a three-bar mechanism in two particular cases for which some elements (position, length, angular velocity, angular acceleration of some links) are given.

For each problem numerical examples are treated; the obtained results are checked with graphical methods; the Grashof condition is controlled. D. DeMeulemeester, Belgium

2424. Technical studies on mechanisms (Lectures held in Dresden at a symposium) (in German), *Maschinenb.-Tech.* **6**, 2, G1-G26, Feb. 1957.

Dr. F. G. Altmann, Braunschweig, in a paper entitled "On dimension synthesis of space mechanisms" exposes the solution of the following problem: Given four, five or more points in the space, find one or several mechanisms for which the curve described by a point of one of the links, for instance a point of the connecting rod, contains the given points. The proposed solution consists in finding two revolution surfaces (cylinders, cones, hyperboloids, spheres, globoids) whose surfaces contain the given points. These points are, in consequence, situated on the intersection line of the two surfaces. The dimensions of these surfaces (radius of the cylinder, top of angle of the cone) and the relative position of their axes fix the dimensions of the mechanisms sought; during the motion of which a point of a determined link describes a line containing the given points.

Geometrical constructions are proposed for resolving several particular problems: determination of two cylinders, of a sphere and a cylinder, of two cones, of two hyperboloids, whose intersection line contains four given points in space; determination of two globoids containing five points of space. The corresponding mechanisms are deduced. The way to construct a space-slider-crank rotating mechanism is shown, when four positions of the crank and the four corresponding positions of the slider block are imposed.

O. Steiner, Braunschweig, referring to the preceding paper of Dr. Altmann, shows the way to determine the parameters of revolution cylinders whose surface contains three, four or five given points, with the use of series of curves, called VD curves. In his paper "Constructions for revolution cylinders" he chooses a point on the surface of a revolution cylinder whose diameter is considered as the scale unity. This point is taken as the center of a series of spheres whose radii are increasing. The intersections of these spheres with the cylinder are developed in the plane tangent at the cylinder in the chosen point. These developed curves are the VD curves. Graphical solutions are given for finding on the surface of a given cylinder, with the aid of VD curves, the possible positions of three (and four) points whose mutual distance is imposed. The determination of a revolution cylinder, whose surface contains five points with imposed mutual distance, is also given.

Dr. W. Roessner, Magdeburg, in "Construction of dead positions for six-link mechanisms whose fixed member is a binary one" considers a rotating shaft driving a swinging lever. When a great angular displacement or when a quick return of this lever is needed, a four-link mechanism will, generally, not give a satisfactory solution. In such cases, the designer will use a six-bar mechanism. Author studies especially a six-link mechanism for which the two ternary members are not directly connected and for which the fixed member is a binary one.

The force transmission from the shaft to the oscillating member and the positions of the dead stands are studied. The problem of designing such a six-bar mechanism is treated in the case where the following elements are given: length of the fixed member, length of the crank, angular displacement ψ_0 described by the oscillating member and corresponding angle ϕ_0 described by the crank.

The obtained solutions are discussed especially from the point of view of the force transmission. Example: construction of six-bar mechanisms realizing definite angles ϕ_0 and ψ_0 are given.

"Cam mechanisms for counting machines and their practical measurements" by Dr. S. Hildebrand points out the fact that, in counting machines, eight important motions are produced by cams placed on the driving shaft, and shows the importance of these mechanisms for the apparatus. Therefore, series of electrical measurements on cam mechanisms were undertaken by author to find the possibility of increasing without trouble the speed of the counting machines. A follower maintained in contact with a cam by means of a spring was used. The contact, the displacement, and the velocity of the roller were electrically recorded in function of time. The effect of damping was examined. Different profiles, different speeds, different contact forces were used, and the limiting speeds assuring in each case, for the given lift, a real contact between roller and profile were determined.

In a paper entitled "Plane combined cam and connecting rod mechanism Dr. J. Mueller, Iena, considers four-bar linked mechanisms in which the connecting rod is fitted with a profile in contact with a slide or with a swinging lever in order to give them some definite motion. Two cases are considered. The desired motions are taken from an existing four-bar mechanism or the dimensions of the four-bar mechanism are not imposed. Several examples are treated in the two cases.

In the paper "Mechanisms in photographical apparatus" Dr. H. Weise, Köln, reproduces some 84 pictures of a film showing motions of mechanisms used in photographic, kinematic and projection apparatus. The film contains 77 titles, explanatory texts, and drawings for 76 patterns. Time is slackened in proportion varying between 20 and 1000.

D. DeMeulemeester, Belgium

2425. Kukhtenko, A. I., On a class of mechanisms with nonholonomic couplings (in Russian), *Trudi Sem. teor. Masb. Mekb.* **15**, 58, 46-70, 1955.

The use of Lagrange equations for analysis of motion of mechanical assemblies with a constant or variable reduced inertia moment was suggested by Artobolevski. These equations cannot be directly applied to mechanisms with nonholonomic couplings. Hence either transformed equations with undetermined coefficients, like Verrers equations, or methods evolved by Appel and Chaplygin must be used.

The mechanisms analyzed in this paper can be used in computing devices or in automatic control systems. The analysis starts with classical example of a sphere on a rough surface. Lagrange equations are analyzed jointly with auxiliary system for nonholonomic constraints, and the attempts to reduce their number to no more than the number of degrees of freedom are pointed out. For their analysis the generalized coordinates and velocities may be replaced by pseudo-coordinates and velocities; their judicious choice may eliminate the undetermined coefficients. Purely mathematical description of constraints may become inadequate when the nature of physical interaction is important. Additional sources of energy are also included in servomechanisms. System of automatic control for a channel-cutting machine is analyzed as problem involving nonholonomic couplings. The undetermined coefficients are used and solution compared to that obtained by using classical Lagrange equation.

P. Bielkowicz, USA

2426. Gortinskii, V. V., Kinematics of inertia brushes moving around in sifter sieves which move in a continuously circular and rectilinear back-and-forth direction (in Russian), *Trudi Sem. teor. Masb. Mekb.* **15**, 58, 5-18, 1955.

Mechanism for sorting ground grain is analyzed and equations of motion of its elements are developed. Relative motion of brushes is caused by the translation of frame or guide. An attempt is made

to rationalize their theory of motion in order to improve the design. The inertia of elements in translational and relative motion, friction and side reaction of guides are analyzed and introduced into equations. Reciprocating motion of sieves is investigated and maximum relative velocity of brushes determined. The results are presented in graphs, equations, and tables.

P. Bielkowicz, USA

2427. Karmadonov, A. F., Analysis of the kinematics of movable couplings (clutches) (in Russian), *Sb. statey chelyabinskogo politekhn. in-ta* no. 1, 70–78, 1954; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8089.

The kinematics of rigid contact couplings (plate or cone clutches) are investigated, enabling angular and parallel displacements of the coupled shafts. An elementary, two-link coupling is analyzed, the driving half whereof is represented by a crank keyed on the drive shaft and having a point directed in the sense of rotation, while the driven half is a flat plate attached on the follower shaft in the plane thereof. It is demonstrated that within the limits of an opening angle of $\leqslant 10^\circ$ and a displacement of $\leqslant 0.1$ from the radius of the driving crank the irregularity of rotation of the follower shaft is far more influenced by the parallel than the angular displacements. It is suggested to evaluate the efficiency of the coupling (for a known coefficient of friction) by the theoretical path of the point on the plane during one revolution. Some structural design considerations are discussed.

The conclusions are of limited application, since no account is taken of the rigidity of the element, the structural deviations of the follower plate surface from the plane of the shaft, and the possible change in the radius of contact of the elements in the case when the halves of the coupling are reciprocally enveloping curves.

V. N. Geminov

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2428. Fazekas, G. A. G., Graphical shoe-brake analysis, ASME Fall Meet., Denver, Colorado, Sept. 1956. Pap. 56-F-3, 10 pp.

2429. Hartenberg, R. S., Complex numbers and four-bar linkages, *Mach. Design* **30,** 6, 156–163, March 1958.

2430. Goodman, L. E., and Robinson, A. R., Thermal drift of floated gyroscopes, *J. appl. Mech.* **24,** 4, 506–508, Dec. 1957.

Author studies the influence of the temperature distribution on a floated gyroscope, which gives rise to currents tending to rotate the gimbal, and also determines the rebalance torque which is required to prevent the rotation of the gimbal. Obviously the details which make it impossible for the gimbal to be perfectly centered are unavoidable. The author investigates the influence of such eccentricity. In the first two paragraphs, the case of a centered gimbal kept steady by a rebalance torque is considered. Since the flow is similar and both boundaries are at rest and $(b/R)<l$ (R the gimbal radius), torque L_0 is determined. Further on, the maximum drift rate is determined. In the third paragraph is considered the case of a centered gimbal which is held fixed by a rebalance torque L, and the ratio L/L_0 is determined. Author represents by b the variable thickness of the fluid gap and putting $b = b_0 + e \cos(\theta - \alpha)$, where bo is the mean thickness, e ($\lesseqgtr b$) the eccentricity, he finds that $L/L_0 \cong [1-(e/b_0)^2]/[1+1/2\,(e/b_0)^2]$.

The effect of the eccentricity is to reduce the rebalance torque below the value which it would have were the gimbal perfectly centered. For small ratios e/b_0 the reduction is slight.

It is to be noted that even in the case of the centered gimbal the equations that give the rebalance torque and the maximum drift rate provide close upper bounds also in the case of the eccentric gimbal.

Paper gives principal conclusions concerning the advantages of the floated gyroscopes to which the considerable works of C. S. Draper, W. Wrigley, L. R. Grohe, etc, have contributed. It also provides subjects for theoretical research and practical appliances.

C. Papaioannou, Greece

2431. Ellett, D. M., How to determine mass moments of inertia for irregular parts, *Prod. Engng.* **28,** 5, 184–187, May 1957.

2432. Meyer, H., Contribution to the theory of table roll of paper machines (in German), *ZVDI* **99,** 11, 449–456, Apr. 1957.

2433. Packer, L. S., and Eiss, N. S., Phase-plane graphics—a new approach to teaching dynamics, *Amer. J. Phys.* **26,** 2, 91–103, Feb. 1958.

A motion can be represented graphically in various plots: displacement versus time, acceleration versus time, or by projection of a rotating vector. Authors propose a representation displacement versus velocity to which, as a third coordinate, time may be added, in which case the representation is a three-dimensional one. Authors claim that these representations are an aid for a student, as facilitating mental visualization. They term the method as "phase-plane graphics" or "trajectories." The method is illustrated on uniform motion, on simple harmonic motion, on motion retarded by a constant force, on impact between two elastic bodies, on oscillatory motion with constant friction force, on dissipative rebound of a spring-mounted mass between two surfaces, and on falling mass rebounding from spring-mounted mass. Reference is given to the book of Stoker: "Nonlinear vibrations," which treats similar problems.

K. J. DeJuhasz, Germany

Servomechanisms, Governors, Gyroscopics

(See also Revs. 2425, 2430, 2527, 2668, 2669, 2746, 2777, 2808, 2841)

Book—2434. Letov, A. M., The stability of nonlinear controlled systems (in Russian), Moscow, Gostekhizdat, 1956, 312 pp. + illus; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7224.

Questions relating to the steadiness of nonlinear control systems with one and two controlling elements are examined. The fundamental material for this investigation has been furnished by the work of A. I. Lurié ("Some nonlinear problems of the theory of servomechanisms," Gostekhizdat, 1951; papers in *Prikl. Mat. Mekh.*) and the present author's articles in *Prikl. Mat. Mekh.* and *Avtomatika i telemekhanika.*

Book is divided into eleven chapters and an introduction in which some problems of the theory of stability are surveyed. In particular, geometrical illustrations are given of the first and second theorems of Liapunov's second method, which is the fundamental analytical method used in the book (in regard to rigorous and complete treatment of the subject, the present illustrative survey does not, of course, replace Liapunov's own original and classical exposition).

The first chapter deals with the equations to be analyzed. It is assumed that the perturbed motion of the system with a single controlling element can be described by the following equations:

$$\dot{\eta}_k = \sum_{\alpha=1}^{m} b_{k\alpha}\eta_\alpha + n_k\mu \quad (k = 1\ldots, m)$$

$$V\ddot{\mu} + W\dot{\mu} + S\mu = f^*(\sigma) \qquad [1]$$

$$\sigma = \sum_{\alpha=1}^{m} p_\alpha\eta_\alpha - r\mu$$

in which η_k represents the coordinates of the controlled object, μ coordinate of the controlling element. The controlled objects

are classified according to the characteristic numbers of the matrix $\|b_{k\alpha}\|$ into inherently stable, neutral (indifferent), and inherently unstable, and this classification is consistently applied in all cases analyzed.

The nonlinear functions $f(\sigma)$ satisfy the conditions $f(\sigma) = 0$ for $|\sigma| \leqslant \sigma^*$, $\sigma f(\sigma) > 0$ for $|\sigma| > \sigma^*$; an interruption of continuity being permitted for the point $\sigma = \pm\sigma^*$ (Class A). (It is not clear from the footnote on page 35, whether $f(\sigma)$ is a steady function σ, or whether, during the control process, this function may vary, when the equation of the process must incorporate the function $f(\sigma, t)$. In the latter case, obviously, methods of analysis disregarding the relationship between f and t, must be inapplicable).

The problem of the absolute steadiness of a system (A. I. Lurié), is formulated; i.e., the problem of steadiness in the presence of any perturbation, at which the equation of the process remains valid, and any functions $f(\sigma)$ of Class A or any subclass thereof.

The following chapters examine the transformation of the system of equations [1] and those analogous thereto, into particular, canonical forms which are then analyzed by Liapunov functions of a special form. These analyses are in great measure founded on and supported by the ideas of A. I. Lurié set forth in his mentioned works.

Steadiness criteria, claimed to have been simplified by the author, are also presented; in particular, a method of constructing Liapunov functions (and modifications of this method) is investigated, already proposed by I. G. Malkin [*Prikl. Mat. Mekh.* **15**, no. 1, 1951], for systems of the kind here investigated.

Chapter VII discusses the question of program control. It is demonstrated that under specific conditions a controlling element maintaining steady-state conditions may be capable of similarly maintaining other steady-state conditions with the same tuning, provided the latter sufficiently closely resemble the former.

In the chapters following, equations of the type [1] are analyzed by Liapunov's second method in regard to the quality of control (from the aspect of rate of decay of stability) under continuously acting perturbations, and steadiness generally in first approximation. A brief survey is made of the stability of particular unsteady motions.

It should be remarked that here (chap. VII, VIII, X, XI) it is insufficiently indicated what relationships subsist between the results obtained in the present book for equations of the type [1] and the results of the general theory of stability; thus, for instance, the connection is not made clear between the preservation of stability and the presence of a small programming force and the general theory of the small parameter.

In chap. IX some results are generalized for the case of systems with two controlling elements.

The methods suggested are illustrated on a number of examples of differential equations for control systems.

N. N. Krasovskii

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2435. Plunkett, R., The calculation of optimum concentrated damping for continuous systems, ASME Ann. Meet., New York, N. Y., Dec. 1957. Pap. 57-A-93, 6 pp.

If concentrated damping is applied elsewhere than at the point of vibratory force there is an optimum value for the damping. Author makes an analysis and applies it to some simple cases. Some general conclusions may be drawn.

F. I. Niordson, Sweden

2436. Popov, E. P., Use of the harmonic linearization method in automatic control theory (in Russian), *Doklady Akad. Nauk SSSR (N.S.)* **106**, 2, 211-214, Jan. 1956.

Krylov-Bogolyubov's harmonic linearization method is applied to automatic control theory. The problem of free motion of large class of nonlinear systems of automatic control governed by Eq. [1]: $Q(p)x + R(p)F(x, px) = 0$, $p \equiv d/dt$, is discussed. Here $Q(p)$, $R(p)$ are polynomials, and $F(x, px)$ the given nonlinear function of general character. It is assumed that Eq. [1] has either a periodic solution or a solution which is very near to periodic, $x = x^* + \epsilon y(t)$, $x^* = a \sin \omega t$, where ϵ is a small parameter; $y(t)$ is bounded function of time.

The form of parameter ϵ is so determined that the method of harmonic linearization can be used. In this way Eq. [1] can be explained in new form: $Q(p)x + R(p)(q + q_1\omega^{-1}p)x + \epsilon f(x, p) = 0$; q and q_1 are integral forms of function $F(a \sin u, a\omega \cos u)$, $u = \omega t$ in region $[0, 2\pi]$, but $\epsilon f(x, p)$ is presented in form of infinite series. [See AMR 8 (1955), Revs. 1578, 3305, and 3306.]

D. Raskovic, Yugoslavia

2437. Kuzovkovi, N. T., An investigation of the stability of systems containing undamped vibrational links, by the method of logarithmic frequency characteristics (in Russian), *Uch. zap. Mosk. un-ta* no. 172, *Mechanics* **5**, 207-213, 1954; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8059.

The possibility is indicated of applying the Nyquist hodograph, and modifications thereof, to systems containing undamped, vibrational links (a pair of purely imaginary roots). By suitably tracing the poles of the transitional function located on the imaginary axis, it is possible to construct a phase-amplitude hodograph, the orientation whereof to the point (0-1) in the usual manner forms a criterion of stability. An example is examined of the steadiness of a gyro-stabilized platform, in the equations of motion whereof, the terms corresponding to viscous friction are eliminated.

L. A. Rozenberg

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2438. Feldbaum, A. A., The problem of the synthesis of optimum automatic control systems (in Russian), Trans. Second All-Soviet Congress on the Theory of Automatic Controls, Vol. 2, Moscow/Leningrad, Akad. Nauk SSSR, 1955, 325-360; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8062.

The problem is examined of the synthesis of an optimum servo system insuring the most rapid development of the transmission process. The problem consists in selecting the structural schematic and determining the parameters of the system to be coupled in front of the invariant part in order to enable the system, as a whole, to eliminate detuning between a given impact value and the output value in the shortest possible time. Since the control system is essentially nonlinear, it must act on the invariant part in such manner that the derived output value or combination of values, subject to control, will in consecutive time intervals assume either one of two limiting values. The number of such transitions will equal the order of the controlled derivative, while the instants of transition will depend on the initial conditions of the system and the variational law of the input variable. If this law is originally unknown, the most probable developments of the process can be predicted from a hypothesis of a statistical character or obtained by extrapolating functions of a previously determined class.

The practical realization of the required system is based on two suggested principles. Applying the first of these, the system requires an incorporated block element solving the nonlinear relationships between the derivatives and the values of the instantaneous transitions; when using the second method, it is first necessary to determine and introduce into the corresponding block element, such a combination of coordinates as will change sign at the instant when the transition (reversal) is to be performed. The examples presented embody schematics in which both methods are used for the case of limitation of the second derivative and a parabolic variation of the input value.

Means are given for synthesizing systems in the presence of directly or indirectly measurable, influencing disturbances. The direction of further developments of the problem is indicated.

I. B. Chelpanov
Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

2439. Fateyev, A. V., A method of graphical analysis for the selection of the correcting feedback links in automatic systems (in Russian), Trans. Second All-Soviet Congress on the Theory of Automatic Control, Vol. 2, Moscow/Leningrad, Adad. Nauk SSSR, 1955, 266–297; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8066.

When calculating the parameters of correcting feedback links introduced into a control system for the qualitative improvement of the transitional processes, the phase-amplitude characteristics of the system and its elements can be utilized.

From the known values of the qualitative constants and the reserve stability, nomograms are plotted to produce a material, frequency characteristic of the corrected system, approximated by three trapezoids. From this characteristic, the imaginary frequency characteristic and the phase-amplitude characteristic of the corrected system are determined.

If the phase-amplitude characteristic of the uncorrected system is already known, a method of graphical analysis can be applied to the construction of the phase-amplitude characteristic of the correcting arrangement, from which its type and parameters can be determined.

Tables are given of typical links and feedback elements, and the necessary nomograms. The calculation is explained on an example of the selection of the controlling parameters of an electric transmission.

V. A. Tropitskii
Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

2440. Tsai, D. H., and Hogue, E. W., Reproducibility of a pen-and-chart type of recording instrument under dynamic conditions, ASME Ann. Meet., New York, N. Y., Dec. 1957. Pap. 57-A-209, 21 pp.

Response to + step (and return) form of input signal has been recorded on commercial instruments, specifically two identical EA units for recording frequency. Repeated measurements of response time of one unit indicated reproducibility within ±4% under best conditions of the experiment. Authors varied pen pressure, chart speed, type of ink, ink level, humidity, temperature, pen attitude and input voltage, and found pen pressure was primary factor in reproducibility of response time. Length of response time was appreciably affected by pen pressure and by torque and damping of the unit, but was less sensitive to chart speed and other factors. Chart speed ranged up to 12 in./min. or 10% of writing speed. Second unit, when made equal to first in torque, magnetic damping, pen pressure, and chart speed, gave identical response to step function and to arbitrary forcing functions.

Reviewer notes an interesting result that is not discussed, that + step always had shorter response time than that of return, and that with excess pen pressure + step was always less reproducible.

D. Moseley, USA

2441. Izawa, K., On-off control with periodic sensing device, ASME Ann. Meet., New York, N. Y., Dec. 1957. Pap. 57-A-207, 17 pp.

Method to control the temperature in a plant is valid for a transfer function consisting of a dead time plus a first-degree lag. Method seeks to pick up at once temperature plus a derivative term (not the same for "On" and "Off" operation) and to actuate the "On Off" heater switching through a periodical sensing circuit. Assigned temperature seems constant, and there is a disturbance θd at the input of the plant. An irregular delay between the relay and the plant is added. Computations use phase plane diagram.

Important results of method are to decrease fluctuation of controlled temperature, when θd varies slowly with respect to switching.

Reviewer does not clearly understand whether random dead times after the switch are a part of given disturbances or a part of synthetized control device. Author does not clearly state the usefulness of switching, and reviewer believes it is some kind of linearizing device. More emphasis is put on description of the actual device than on reasons of its choice.

J. M. Loeb, USA

2442. Segel, L., Research in the fundamentals of automobile control and stability, *SAE Trans.* **65,** pp. 527–540, 1957.

Paper defines "handling," "control" and "stability" as applied to automobile, summarizes pre-1956 literature, develops lateral equations of vehicle motion, describes experimental verification and uses the theory so derived to explain basic steady and transient car-handling characteristics.

Author remarks on paucity of data on tire mechanics and autoaerodynamics and lack of knowledge of characteristics wanted and needed by drivers. Further information on these points is promised. Some of this is to be found in five subsequent papers by author and his colleagues; "Research in automobile stability and control, and in tyre performance" *Instn. mech. Engrs., Auto. Div.* 13 Nov. 56, which deal more fully with the problem.

E. M'Ewen, England

2443. Kalisch, G. G., A quantitative evaluation by the Vyshnegradsky diagram of the convergence of the process and the resonance phenomena in the automatic control of internal combustion engines (in Russian), Trans. Second All-Soviet Congress on the Theory of Automatic Control, Vol. 1, Moscow/Leningrad, Akad. Nauk SSSR, 1955, 567–573; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8063.

A method is presented for evaluating the quality of direct-governing systems for internal-combustion engines, by means of the well-known diagrams of I. A. Vyshnegradsky. These diagrams give curves of equal convergence and divergence of the governing processes and curves of equal vibrational frequency in the region of vibratory motion of the governing system.

The convergence of an aperiodic process is characterized by the time required for the initial deviation to decrease by half. The convergence of the vibratory process is determined by the damping decrement.

The diagrams of Vyshnegradsky also determine the region of resonance arising in a governing system by the action of the periodic variation of the engine torque. For this region, two families of curves are given, corresponding to equal frequencies of the resonance vibrations and equal values of the coefficients of intensification of the periodic disturbances of the system.

The paper is a development of the work of M. A. Izermann [*Avtomatika i Telemekhanika* no. 1, 55–65, 1940].

S. I. Bernshtein
Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

2444. Adler, F. P., Missile guidance by three-dimensional proportional navigation, *J. appl. Phys.* **27,** 5, 500–507, May 1956.

Author extends usual definition for planar proportional navigation to three dimensions, obtaining two equations for desired missile turning rates. One is the planar case, the other the turning rate normal to reference plane. Combining these with missile-target geometry, author derives basic linearized equations of motion for disturbances of missile from reference path during

tracking. By neglecting time-dependence of time-to-go parameter, author can integrate these equations once. Final equations replace arbitrary integration constants by initial conditions on missile aiming error and target displacement from reference path. Missile velocity, closing velocity, and guidance gain are shown to be important parameters of motion. Example calculations are shown to give good agreement with exact solutions for missile transfer function $Z(p) = 1$. A. A. Schy, USA

2445. Dyachenko, M. Ya., The equation of motion of a hydrodynamic turbine coupling with automatically controlled slip (in Ukrainian), *Dopovidi Akad Nauk URSR* no. 2, 118–122, 1955; *Ref. Zb. Mekh.* no. 12, 1956, Rev. 8072.

An attempt is made to describe the processes taking place in a turbine coupling with controllable slip by means of an ordinary differential equation of the first order. M. A. Aizerman

Courtesy Referativnyi Zburnal, USSR

Translation, courtesy Ministry of Supply, England

2446. Grammel, R., and Ziegler, H., High-speed symmetrical gyro in Cardan support with friction (in German), *Ing.-Arch.* **24**, 6, 351–372, 1956.

A method is given for the analysis of the motion of a high-speed gyro as influenced by dry and also viscous friction in the support bearings. Motion under the action of a gravitational moment and without such moment is given. It is found that the presence of friction increases precisional velocities. The results were verified experimentally to a fair degree of accuracy.

E. A. Trabant, USA

2447. Mikhalev, N. A., The theory of the rigorous gyroscopic vertical and an arrangement for compensating velocity deviations (in Russian), Elements of the calculation of precision instruments, Moscow, Oborongiz, 1954, 97–111; *Ref. Zb. Mekh.* no. 12, 1956, Rev. 8019.

An arrangement for a vertical gyroscope with correcting means for velocity deviations (precessional movements) is described and discussed.

Author develops the equations of motion of the instrument and analyzes them for the case of displacement of the support of the vertical gyroscope along the surface of the earth at a constant speed; it is demonstrated that the vertical gyroscope will not experience precessional deviations. Eq. (14) on p. 104, contains typographical errors. A. M. Letov

Courtesy Referativnyi Zburnal, USSR

Translation, courtesy Ministry of Supply, England

2448. Danilin, V. P., The classification and analysis of gyroscopic horizons (in Russian), *Trudi MAI* no. 50, 142–172, 1955; *Ref. Zb. Mekh.* no. 12, 1956, Rev. 8018.

A classification is presented of gyroscopic assemblies, consisting of a gyroscope with correcting means, returning the gyroscope to the vertical.

Author classifies such gyroscopic horizons according to the principle of action of the correcting means. Accordingly, he distinguishes gyroscopic horizons with radial correction (following the terminology of B. V. Bulgakov in his book "Applied gyroscope theory," 1939, p. 64); the gyroscopic horizons with "gyropendulum correction." (It would seem preferable to classify such instruments as "gyro-horizons with conical correction," considering, as in the first case, the geometrical pattern of the motion of the axis of the gyroscopic configuration.)

Within each of these groups, author divides gyro-horizons into distinct subgroups, classified according to the aspect of the function determining the relationship between the correcting moment and the angle of inclination of the gyroscope.

Trajectories are given for the motion of the apex of the gyroscope, when carried on a fixed support, for the gyroscopes included in the classification.

Author's statement that, for new types of gyroscopic appliances, instead of "analyzing the frequently very complex physical pattern of the work of the correcting means, it is sufficient to determine experimentally the pattern of the trajectories of the apex of a gyroscope on a fixed support, and the velocity of motion of the apex along each trajectory, from which data the particular instrument is located in the classification, and such instrument evaluated according to its corresponding group in the classification, with regard to its behavior in the maneuvering of a particular ship or aircraft," appears to be at least disputable.

Ya. N. Roitenberg

Courtesy Referativnyi Zburnal, USSR

Translation, courtesy Ministry of Supply, England

2449. Tikhmenev, S. S., Azimuthal precessions of a semi-gyroscopic compass due to perpendicular corrections of the gyroscope axis with reference to the axis of the external gimbal ring in the presence of oscillations of the instrument about some horizontal axis (in Russian), Elements of the theory and calculation of gyroscopic and navigational instruments (MVTU **48**), Moscow, Oborongiz, 1955, 37–56; *Ref. Zb. Mekh.* no. 12, 1956, Rev. 8020.

Various cases are examined of the harmonic oscillation of an instrument: about a single axis, simultaneous oscillations about two axes with identical frequency and phase-shift; simultaneous oscillations about two axes with different frequencies. The gyroscope is acted upon merely by a correcting moment proportional to the angular deflection of the inner gimbal ring. The differential equations determining the precessional motion of the gyroscope axis are integrated on the assumption of small amplitude of the oscillations of the instrument. As a result, the author, isolating the periodically variable terms, obtains expressions determining the mean value of the angular velocity of azimuthal precession of the gyroscope. These formulas show that steady equilibrium positions of the gyroscope exist.

The paper contains misprints. V. N. Skimel

Courtesy Referativnyi Zburnal, USSR

Translation, courtesy Ministry of Supply, England

Vibrations, Balancing

(See also Revs. 2405, 2418, 2419, 2435, 2436, 2437, 2483, 2568, 2832, 2905, 2909)

2450. Foss, K. A., Coordinates which uncouple the equations of motion of damped linear dynamic systems, ASME Ann. Meet., New York, N. K., Dec. 1957. Pap. 57-A-86, 4 pp.

Orthogonality relations between the eigenvectors of damped linear dynamic systems with lumped parameters are derived; and from these relations coordinates are found in terms of which uncoupled equations of motion can be written. Methods are developed for determining transient stresses in terms of these coordinates. The present treatment is extended to systems involving transient damping and to continuous systems.

From author's summary by E. J. McBride, USA

2451. Copp, Yu. A., Linearization of the restoring force by the method of piecewise linear approximation (in Russian), *Inzbener. Sbornik, Akad. Nauk SSSR* **18**, 149–152, 1954.

The problems of nonlinear vibrations in engineering practice are frequently solved by means of equivalent linearization [for the mathematical foundation of such a procedure see Kriloff-Bogoliu-

boff, "Introduction to nonlinear mechanics," Annals of mathematics studies, no. 11, Princeton University Press 1943, 2nd ed. 1947]. Some well-known methods of linearization of the restoring force are compared in this paper with the method of piecewise linear approximation proposed by the author.

D. Radenkovic, Yugoslavia

2452. Sokolov, Yu. D., Determination of dynamic forces in mine-shaft hoisting cables (in Russian), *Prikl. Mekh.* **1**, 1, 23–35, 1955; *Ref. Zb. Mekh.* no. 12, 1956, Rev. 8556.

A method is adopted, put forward earlier by the author, of applying to the approximate solution one differential equation in particular derivatives, describing the longitudinal oscillations of a mine shaft hoisting cable, deduced by G. N. Savin. The problem allies itself to the integration of the system of two ordinary linear heterogeneous equations with rational coefficients.

From this system an approximate linear differential equation of the fourth order is deduced to determine the dynamic force in the lower end of the cable. Two variants (transformations) appear, which are useful in the approximate solution. The question is discussed on the behaviour of the calculations of the system of two equations deduced by the author in the case when the length of the cable $l = l(t)$ is tending towards zero.

V. N. Shevelo

Courtesy Referativnyi Zburnal, USSR

Translation, courtesy Ministry of Supply, England

2453. Yashchenko, O. A., The oscillations of an elastic filament with a variable end mass (in Ukrainian), *Dopovidi Akad. Nauk URSR* no. 6, 529–532, 1955; *Ref. Zb. Mekh.* no. 11, 1956, Rev. 7793.

A solution by the method of characteristics is given for the problem of the longitudinal oscillations of an elastic filament, the upper end of which is attached, whereas, to the lower end, a vessel of mass m_0 is suspended which, at a particular instant of time, commences to fill with an additional mass by a linearly time-variable law.

M. E. Temchenko

Courtesy Referativnyi Zburnal, USSR

Translation, courtesy Ministry of Supply, England

2454. Mahalingam, S., Polygonal action in chain drives, *J. Franklin Inst.* **265**, 1, 23–28, Jan. 1958.

Paper is concerned only with dynamic load induced in chain drives by polygonal action of sprocket and specifically excludes consideration of chain surge which dominates high-speed chain action. Author develops equation for dynamic load in chain in terms of number of sprocket teeth, longitudinal stiffness of chain, speed ratio, phase angle between sprockets, natural frequency of system, and frequency of tooth engagement. No damping is taken into account nor is the nonlinear stiffness of chains. The obvious result is obtained that large forces may develop when the frequency of tooth engagement is equal to the natural frequency of the system, but the difficulties of establishing the latter are not treated.

Within limitations of the premises, author shows that polygonal dynamic load is a minimum with an integral number of pitches in the common tangent (and zero if speed ratio is then unity) and a maximum with an odd number of half pitches in the common tangent.

Reviewer considers paper of little practical utility but is glad to see author's interest in field which has not been adequately studied.

E. M'Ewen, England

2455. Gere, J. M., and Lin, Y. K., Coupled vibrations of thin-walled beams of open cross section, ASME Ann. Meet., New York, N. Y., Dec. 1957. Pap. 57-A-26, 6 pp.

Coupling between bending and torsional vibrations leads to a system of three fourth-order differential equations. Related auxiliary algebraic equation is of 12th degree. Application of boundary conditions leads to a frequency equation, which authors solve numerically on an IBM 605 Calculator. Approximate solutions by Rayleigh-Ritz method are compared with these results and show good agreement. Since all end conditions show very much the same effect of coupling, authors suggest finding uncoupled frequencies for actual end conditions and effect of coupling from the case of simply supported ends, which is more easily treated.

F. I. Niordson, Sweden

2456. Volterra, E., and Zachmanoglou, E. C., Free and forced vibrations of straight elastic bars according to the "method of internal constraints" (in English), *Ing.-Arch.* **25**, 424–436, 1957.

Using assumed functions of the type

$$u(x,y,z;\ t) = v_x(x,t) + y\lambda_x(x,t) + z\mu_x(x,t)$$

for the components u, v and w of the elastic displacements, the equations of motion and boundary conditions are deduced, where the effects of shear and rotatory inertia are also taken into account. Supposing that all functions entering into the assumed function contain explicit functions of the coordinate axes and the time, solutions are found for free and forced lateral vibrations of simply supported, cantilever, built-in, and free-free bars, as well as for bar built in at one end and simply supported at the other. For bars of circular cross section, solutions are deduced in a similar way for the longitudinal vibration of pinned-end, cantilever, built-in, and free-free bars as well as for bars built-in at one end and pinned at the other. Finally, a similar solution is given for torsional vibration.

S. T. A. Odman, Sweden

2457. Klotter, K., and Kreyszig, E., About a special group of self-excited vibrations (in German), *Ing.-Arch.* **25**, 389–403, 1957.

Vibration is mathematically presented by

$$\ddot{q} - (\operatorname{sgn}\dot{q})\beta/2(1-\alpha^2q^2)\dot{q}^2 + \kappa^2 f(q) = 0$$

where $\beta > 0$, $\alpha \neq 0$ and $f(q)$ is an arbitrary function. Introducing $V = \dot{q}^2$, equation takes the form

$$dV/dq - (\operatorname{sgn}\dot{q})\beta(1-\alpha^2q^2)V + 2\kappa^2 f(q) = 0$$

This equation is discussed in detail. Its physical interpretation and various types of origin boundary conditions are also considered. Limitation of the amplitude is developed, as well as the existence and the lower and upper limits of the limit amplitude.

Some results of numerical calculations are given in diagrams. An amplitude diagram is calculated for $f(q) = q$ and $\alpha/\beta = 1$, and the dimensionless limit amplitude is given as a function of α/β for $f(q) = q$ and for $f(q) = q^3$. Finally, an approximate solution is given given, which is valid for low values of β and $f(q) = q$.

S. T. A. Odman, Sweden

2458. di Taranto, R. A., Effect of elastic racks on vibration mounting of equipment, *J. appl. Mech.* **24**, 4, 617–620, Dec. 1957.

The over-all natural frequency of a uniform beam supported at each end is determined in terms of the first mode bending natural frequency of the beam and the natural frequency of the lumped mass beam on the two springs.

From author's summary by M. Rand, Canada

2459. Schaechter, O. Ya., The relationship between the amplitude of constrained, vertical vibration of a pile or sheet pile and the characteristics of the vibrator (in Russian), *Trudi n.-i. in-ta osnovaniy i fundamentov* no. 27, 58–79, 1955; *Ref. Zb. Mekh.* no. 11, 1956, Rev. 7197.

The vertical periodic oscillations of a round pile or sheet pile are investigated, induced by a vibrating driving element; the process of sinking of the driven pile accompanying these oscillations

is disregarded. The motion of the pile is described by the equation

$$m\ddot{z} + R_1(z) + R_2(\dot{z}) = M_0 \epsilon\, \omega^2 \sin(\omega t + \varphi_0) \qquad [1]$$

in which m = mass of the system, m_0, ϵ and ω the mass, eccentricity and angular velocity of rotation of the unbalanced loads of the vibrator, respectively, φ_0 constant phase shift. Essentially, four different laws of change of the soil resistance forces R_1 and R_2 are to be considered:

$$(1)\ R_1 = a\left(z - \frac{4}{3}bz^3 + \frac{4}{5}bz^5\right),\ R_2 = \alpha\dot{z};$$

$$(2)\ R_1 = kz,\ R_2 = \beta\dot{z}^p;$$

$$(3)\ R_1 = kz,\ R_2 = F\ \mathrm{sign}\ \dot{z};$$

$$(4)\ R_1 = \begin{cases} cz + Q_0, & z > 0 \\ Q_1, & z < 0 \end{cases};\ R_2 = 0$$

where a, b, c, α, β, k, p, F, Q are constants

In cases (1) and (2), the periodic solutions of Eq. [1] corresponding to the steady, constrained vibration of the pile were found approximately; for the remaining cases, rigorous solutions are given, it being additionally assumed in case (3), that

$$\dot{z} < 0 \text{ for } 2\pi n < \omega t < \pi(2n+1)$$

$$\dot{z} > 0 \text{ for } (2n+1)\pi < \omega t < 2(n+1)\pi$$

where $n = 0, 1, 2, \ldots$; while for case (4), it is assumed that the period of oscillation is equal to the period of the perturbing force: $2\pi/\omega$. The expressions obtained have been used for the construction of resonance curves which are then compared with the experimentally determined relationships. The experiments were made with a vibrator weighing 700 kg and a static moment of the eccentrics, $m_0 g\epsilon$ up to 660 kgcm, the vibrator rotating at up to 1700 rpm and the pile diameter varying from 12 to 31 cm. It has been found that, at comparatively low values of $m_0 g\epsilon$, the experimental values of the amplitude of vibration agree well with some calculated by a linear law of soil resistance; for relatively large values of $m_0 g\epsilon$ it is necessary to apply a nonlinear law of (soil) resistance.

The limiting values of the amplitude of oscillation of the pile, obtained experimentally, at sufficiently high values of ω are within 0.6–0.8 of the theoretical values. I. I. Blekhman

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2460. Hoppmann, W. H., II, and Magness, L. S., Nodal patterns of the free flexural vibrations of stiffened plates, *J. appl. Mech.* **24,** 4, 526–530, Dec. 1957.

A study is made of the nodal patterns of a particular plate with orthogonal stiffeners. Frequencies of flexural vibration are given for theory and experiment. Good agreement is found.

From authors' summary by M. Rand, Canada

2461. Kato, T., Fukita, H., Nakata, Y., and Newman, M., Estimation of the frequencies of thin elastic plates with free edges, *J. Res. nat. Bur. Stands.* **59,** 3, 169–186, Sept. 1957.

Authors present a variational method for the approximate determination of the frequencies of thin elastic plates with free edges. The procedure allows the determination of both *upper and lower bounds* of the eigenvalues, thus allowing a rigorous estimation of the error.

The method is based on a theorem concerning a self-adjoint operator in Hilbert Space which was proved by T. Kato. After a statement and discussion of this theorem, the operator is used to derive the authors' procedure. Numerical results are obtained for the upper and lower bounds of the fundamental frequency of a square plate with free edges, with excellent results. A generalization to more complicated boundary conditions is discussed and the method is also considered in connection with an earlier procedure by Weinstein and Aronszajn. M. L. Baron, USA

2462. Kane, T. R., Reflection of dilatational waves at the edge of a plate, *J. appl. Mech.* **24,** 2, 219–227, June 1957.

Author has extended work on high-frequency extensional vibrations of plates; the earlier work is briefly reviewed in present paper. Displacements are described in terms of three independent displacement potentials. Solution indicates dilatational wave that is propagated toward edge at an arbitrary angle of incidence gives rise to two dilatational waves and one shear reflected wave.

Sections are titled: 1. Introduction; 2. Plate displacements, stresses and equations of motion; 3. Straight-crested waves; 4. Incident and emergent waves; 5. Boundary conditions; 6. Normal incidence; 7. Oblique incidence; 8. Generalized plane stress.

Paper is well written. J. W. Mar, USA

2463. Le'derman, Yu. R., Parametric resonance of elastic plates (in Russian), *Dokladi Akad. Nauk UzSSR* no. 3, 9–13, 1955; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8538.

The problem of the stability of a plane condition of deformation of an elastic plate, loaded in the central area by periodic forces, by using Green's function for cross displacements, fits in with integral-differential equations of a known type. Re-solving the solution into a series according to the fundamental functions of the nucleus, author obtains a system of ordinary differential equations with periodic functions. These equations are written out for the case of a rectangular plate freely supported along its contour, the plate being subjected to even compression and also to pure shear. The first problem was examined by V. A. Bodner [*Prikl. Mat. Mekh.* 1938] and V. N. Chelomee ["Dynamic stability of the elements of aircraft construction," Izd.-vo Aeroflot, 1939].

V. V. Bolotin

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2464. Moshensky, N. L., The critical rigidity of ribs damping the vibrations of a plate (in Russian), *Trudi Leningr. Korablestroit. in-ta* no. 16, 34–37, 1955; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7784.

The critical rigidity of stiffening ribs is determined by the frequency equation of A. S. Lokshin [*Prikl. Mat. Mekh.* **2,** no. 2, 1935]. V. A. Postnov

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2465. di Taranto, R. A., A blade-vibration-damping device—its testing and a preliminary theory of its operation, *J. appl. Mech.* **25,** 1, 21–27, Mar. 1958.

When a hollow rotor blade, as employed with present-day aviation gas turbines, is filled with a strand of parallel wires, a considerable damping of the bending vibrations of the blade is observed, in the rotating as well as in the nonrotating condition. The two mechanisms, transverse movements of the wires (rubbing around each other) and longitudinal movements (sliding along each other), are investigated. The first movement is dominant in the nonrotating state. The second movement is only effective in the rotating condition if, due to the position of the blade with respect to the center of the rotation, a component of the centrifugal force provides sufficiently large pressures between the wires and inner surface of the blade.

The results of a theoretical investigation are in fair agreement with experimental data. R. G. Boiten, Holland

2466. Godzevich, I. N., The determination of the natural frequencies of turbo-generator foundations (in Russian), *Trudi Uralsk. politekhn. in-ta* no. 54, 133–136, 1955; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7941.

For determining the natural frequency of vibration, it is recommended to treat the foundation as a three-dimensional framework consisting of two plates interconnected by a system of elastic struts. The upper plate is capable of deformation into a cylindrical envelope, the lower plate is completely rigidly supported on an elastic base. Such a system possesses 10 degrees of freedom.

E. I. Silkin

Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

2467. Horovitz, M., The suspension of internal-combustion engines in vehicles, Instn. mech. Engrs. Auto. Div., Prepr., 21 pp., 1957.

2468. Broadbent, H. R., Forces on a brake block and brake chatter, *Instn. mech. Engrs. Proc.* **170,** 31, 16 pp., 1956.

Paper gives study of static and dynamic forces acting on a brake block applied to a wheel of a railway vehicle. Test figures of block friction and adhesion are discussed. Consideration is given to a theory of brake chatter or oscillation of a brake block, its carrier, and the brake rigging. Author concludes that brake chatter will be alleviated by a reduction in clearance in pins and bushes in conventional brake rigging. P. G. Jones, USA

2469. Shimanskii, Yu. A., Determination of the transverse oscillation frequency of a propeller shaft (in Russian), Sb. statei po sudostroenniyu, Leningrad, Sudpromgiz, 1954, 324–330; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8637.

The solution of the problem leads to the determination of the frequency of free oscillations of a prismatic shaft supported at intervals, loaded evenly with its own distributed weight and with the concentrated weight of the propeller at one of the ends of the shaft. The oscillation of the system indicated is examined as the aggregate of the free oscillations of all the spans of the shaft, made with the same frequency, while each of these spans is viewed as a double-supported shaft with a positive or negative elastic fastening of its ends. The end of the shaft, carrying the propeller, is substituted by a corresponding cantilever shaft. The general relationships are indicated, linking the frequency of free oscillations of the doubly supported shaft with the coefficients of the stiffness of the fastening of its ends and permitting the use, for solving the problem, of the method of successive approximations. In terms of the first approximation the author recommends the assignment of a value of the sought frequency of free oscillations close to the frequency of oscillations of the longest span of the shaft. Assuming that all the spans of the shaft oscillate at this frequency, it is possible, by going consecutively from one span to the next, to determine the frequency of free oscillations of the terminal cantilever span. If this frequency is significantly different from the initial one, then the whole process has to be repeated. The application of the method is well illustrated by numerical examples. N. N. Babaev

Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

2470. Kunayev, I. P., The evaluation of the experimental curves of fading vibrations (in Russian), Elements of calculation for precision instruments, Moscow, Oborongiz, 1954, 119–125; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8057.

On the assumption of a material system with one degree of freedom, an experimental curve of vibration in the simultaneous presence of a resisting force proportional to the first power of the velocity, and a Coulomb frictional force, is given for the case of fading vibrations. A method is presented for calculating the fundamental characteristics of the aforesaid forces by measuring the sums and differences of the amplitudes of a particular experimental curve. The fading of the vibration is then examined in the presence of only a resisting force proportional to the first power of the velocity, or of only a Coulomb frictional force, acting on the material system. G. N. Bukharinov

Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

2471. Miles, J. W., Ring damping of free surface oscillations in a circular tank, ASME Ann. Meet., New York, N. Y., Dec. 1957. Pap. 57-A-31, 3 pp.

Sloshing oscillations of a liquid in a cylindrical tank may be damped by means of a submerged annular ring parallel to the free surface of the liquid and rigidly connected to the wall of the tank. It is shown from theoretical considerations and confirmed by experiments that the logarithmic decrement of the damping is roughly proportional to the square root of the amplitude of the sloshing oscillation and the three-halves power of the ring area.

The device proposed seems useful for a number of applications, and a further study of its behavior and the most effective design should be recommended. R. G. Boiten, Holland

2472. Stewartson, K., Toroidal oscillations of a spherical mass of viscous conducting fluid in a uniform magnetic field, *ZAMP* **8,** 4, 290–297, July 1957.

Wave Motion in Solids, Impact

(See also Revs. 2488, 2568, 2573, 2587, 2608)

2473. Sternberg, E., and Eubanks, R. A., On stress functions for elastokinetics and the integration of the repeated wave equation, *Quart. appl. Math.* **15,** 2, 149–153, July 1957.

A completeness proof for the generalized Galerkin solution of the field equations in elastokinetics is supplied and is related to Papkovich and Neuber general solutions of the equilibrium equations. Due to the fact that the Galerkin vector in elastokinetics, in the absence of body forces, satisfies a repeated wave equation, it is further proved that every solution of such an equation may be represented as the sum of two wave functions. This last result generalizes the well-known theorem of E. Almansi regarding the integration of the biharmonic equation.

E. Volterra, USA

2474. Arzhanykh, I. S., The functions of the dynamic stress tensor (in Russian), *Trudi Sredneaz. un-ta* no. 37, 163–172, 1954; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7770.

Different methods are examined of introducing stress functions in dynamic problems of the theory of elasticity, hydrodynamics, and gas dynamics. A preliminary communication on these researches appeared in *Dokladi Akad. Nauk SSSR* **83,** no. 2, 1952.

N. A. Kil'chevskii

Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

2475. Denissyuk, I. N., The polynomials in the problem of tensile impact (in Russian), *Ukr. matem. zh.* **6,** 4, 423–429, 1954; *Ref. Zh. Mekh.* no. 10, 1956, Rev. 6944.

The problem of the so-called tensile impact is examined, i.e., the application of a load with a particular velocity to the lower end of an elastic bar (filament).

This problem is analyzed in application to the braking line of a rope parachute; the displacements in the braking line under tensile

impact are expressed in Laguerre polynomials. Some properties of linear combinations of these polynomials are discussed.

V. N. Shevelo
Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

2476. Komarov, M. S., and Stolyarchuk, V. F., Determination of the dynamic loading on a hoist rope during starting (in Russian), *Nauch. zap. Lvovsk. politekhn. in-ta* no. 26, 3-12, 1955; *Ref. Zh. Mekh.* no. 10, 1956, Rev. 7084.

The dynamic forces in the hoisting and tail ropes of a hoist (lift) are determined. The starting moment for hoisting the load is discussed. The weight of the ropes and change in length of the tail rope, and their rigidity at starting, are neglected. The rotating parts of the hoist are regarded as an equivalent mass M_1, referred to the radius of the rope drum. The excess force F by which the system is set in motion, and which is applied to M_1, is regarded as constant.

On these assumptions, the problem is resolved into the solution of a system of three differential equations of the second order with constant coefficients. As a result, comparatively simple expressions are obtained for the forces in the hoisting and tail rope. In the particular case considered the stress in the hoisting rope (correcting the misprints) is determined by the expression:

$$F_2 = \frac{Fc_2}{m_1(K_1^2 - K_2^2)}\left[\frac{1}{K_2^2}\left(\frac{c_2}{m_2}\frac{c_2 + c_3}{m_1} - K_1^2\right)\cos K_2 t - \frac{1}{K_1^2}\left(\frac{c_2}{m_2} + \frac{c_2 + c_3}{m_1} - K_2^2\right)\cos K_1^t\right] + \frac{Fm_3}{m_1 + m_2 + m_3} + Q \quad [*]$$

in which K_1 and K_2 are the roots of the characteristic equation, m_2 and m_3 the masses of the load and counterweight respectively, c_2 and c_3 the rigidities of the hoisting and tail rope respectively, and Q the ultimate weight of the load.

Analyzing this expression, the authors find two possible maximum values of the forces in the rope, at $\cos K_1 t = \cos K_2 t = -1$, and at $\cos K_2 t = -\cos K_1 t = 1$.

It must be noted that determination of the maximum stress by these means may prove to be impossible if (1) the ratio K_1/K_2 is an irrational number (when $\cos K_1 t$ and $\cos K_2 t$ cannot simultaneously assume the value of ± 1 at any instant of time t); (2) the ratio $K_1/K_2 = n_1/n_2$ is a rational number, and n_1, n_2 are odd integers, while the coefficients of the cosines in Eq. [*] are of opposite sign; (3) the ratio $K_1/K_2 = n_1/n_2$ has one of the integers n_1 or n_2 odd and the other even, while the coefficients of the cosines are negative.

I. A. Prusov
Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

2477. Postol'nik, Yu. S., Dynamics of a ponderable elastic viscous thread of variable length loaded at the end (in Russian), Avtorefer. diss. kand. fiz.-matem. nauk Kievsk. gos. ped. in-ta, Kiev, 1956; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8561.

2478. Robinson, A., Wave propagation in a heterogeneous elastic medium, *J. Math. Phys.* **36,** 3, 210-222, Oct. 1957.

Author considers waves possessing a discontinuity (e.g., of material velocity) across wave front in isotropic medium with elastic constants variable in space. Conclusions include: that a sharp longitudinal wave cannot be created by gradual transformation from a sharp transverse wave (and conversely), and that the relation between the two components of a sharp transverse wave is governed by a law depending on the torsion of the ray. Author believes conclusions are applicable to seismic shocks when variation of the wave in space is large relative to variation of material properties.

Part of the analysis is made for a system of N linear second-order partial differential equations for N functions of n coordinates and remainder analysis is for the case $N = 3$ (displacement components) and $n = 4$ (three space coordinates and time). Conclusions stated above are for the latter case.

L. Malvern, USA

2479. Babitch, V. M., The application of the Hadamard method to the dynamics of an inhomogeneous elastic medium (in Russian), *Vestnik Leningr. in-ta* no. 1, 107-124, 1956; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7769.

A solution is given for the two-dimensional Cauchy problem of the dynamics of an inhomogeneous elastic medium. The equations of motion are used in the form adopted by S. G. Mikhlin [*Prikl. Mat. Mekh.* **11**, 1947]. By the Hadamard method, the matrix of the fundamental equations is constructed of the functions of S. L. Sobolev [*Tr. Seismol, in-ta Akad. Nauk SSSR* no. 6, 1930] for longitudinal and transverse waves. The properties of these fundamental solutions are investigated, and the fundamental integral identity established. As a result, a system of integral equations for the solution of the Cauchy problem is constructed, and a solution of this system by the method of successive approximations is presented.

I. S. Arzhanykh
Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

2480. Kosminskaya, I. P., Amplitude curves and phase hodographs of waves generated by a concentrated harmonic force in a homogeneous, ideally elastic space (in Russian), *Trudi Geofiz. in-ta Akad. Nauk SSSR* no. 30, 286-301, 1955; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7774.

The solution of the problem of the wave pattern generated in a homogeneous, ideally-elastic space by the action of a concentrated harmonic force is presented in the form of simple analytical functions. Utilizing this presentation, author has constructed, and painstakingly investigated, the amplitude curves and phase hodographs of the oscillations to be observed in different directions at varying distances from the source.

In the vicinity of the source, the construction of the chart assumes a complicated form, while the rate of propagation of the oscillations is practically equal to the velocity of the transverse waves. With increasing distance from the source in the region adjoining the line of action of the force, longitudinal waves predominate, while in the region adjacent to the plane perpendicular to the said line, transverse waves dominate. In the intermediate regions, where the intensities of the longitudinal and the transverse waves are comparable, the phase hodographs and amplitude curves assume an interferential character.

K. I. Ogurtsov
Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

2481. Deresiewicz, H., and Mindlin, R. D., Waves on the surface of a crystal, *J. appl. Phys.* **28,** 6, 669-671, June 1957.

Authors study propagation of plane extensional or flexural waves in crystal plates. As in infinite isotropic elastic plates, they find that surface waves are possible and that there is only one limiting surface mode (and one velocity) for both flexure and extension. Monoclinic symmetry and orthorhombic symmetry are examined. For orthorhombic symmetry, authors prove existence of an x-z face-shear mode and a y-z face-shear mode (x-y plane in surface of plate). The y-z face-shear mode yields usual formula for frequency of vibration in a plate of constant thickness. For AT cut of quartz, velocity of the surface wave is calculated. Calculation requires simultaneous solution of algebraic equations of third and fifth degrees as compared to single cubic equation in case of waves of plane strain.

A. P. Boresi, USA

2482. Roy, K. D., Dynamics of the vibration of a bar excited by transverse impact of an elastic load: displacement at any point during impact, *Indian J. theor. Phys.* **4,** 3, 65–70, Sept. 1956.

This is a continuation of the author's earlier work on the same subject. The solution is obtained by Laplace-transform operational methods. The problem considered is that of a cantilever beam struck at an arbitrary interior point by an elastic hammer. Ordinary theory of beam-bending is used, i.e., shear deformation and the like are neglected.

B. A. Boley, USA

2483. Tveritin, A. N., A mathematical analysis of the simplest boundary problem in the theory of longitudinal (axial) impact on an elastically viscous bar with supported ends (in Russian), *Trudi Dnieprostroysk. in-ta inzh. zh.-d. transp.* no. 23, 24–60, 1953; *Ref. Zh. Mekh.* no. 10, 1956, Rev. 6946.

For the equation of the longitudinal oscillations of an elastically viscous bar

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2} + a^2\mu \frac{\partial^3 u}{\partial x^2 \partial t}$$

the following boundary problem is solved

$$x = 0, \quad \frac{\partial u}{\partial x} + \mu \frac{\partial^2 u}{\partial t \partial x} = 0 \qquad (t \geqslant 0)$$

$$x = 1, \quad \frac{\partial^2 u}{\partial t^2} = -H\left(\frac{\partial u}{\partial x} + \mu \frac{\partial^2 u}{\partial t \partial x}\right) \qquad (t > 0)$$

$$t = 0, \quad u = 0$$

$$t = 0, \quad \frac{\partial u}{\partial t} = 0, \quad 0 \leqslant x < 1$$

$$t = 0, \quad \frac{\partial u}{\partial t} = -v, \quad x = 1.$$

Yu. N. Rabotnov

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2484. Dick, J., Shock waves in helical springs, *Engineer, Lond.* **204,** 5298, 193–195, Aug. 1957.

Paper considers the case of a helical spring loaded so rapidly that the coils close against each other and form a "piston" of a portion of the spring. When this occurs there is a shock wave at one or both ends of the "piston" which propagates at greater velocity than the wave velocity given by the classical analysis which assumes that the coils remain open. Shock conditions and wave velocities are calculated. A complete analysis is given for the case of a spring with one end fixed and a square pulse of velocity greater than the classical velocity of propagation applied to the other end.

C. T. West, USA

2485. Kartsivadze, G. N., The construction of an engineering theory of lateral impact (in Russian), *Trudi in-ta stroit. dela Akad. Nauk Gruz SSR* **5,** 73–85, 1955; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7785.

The general system of calculation for lateral impact with consideration of local deformations is discussed, the problem being reduced to an integral equation of the Timoshenko type.

When local deformations are linearly related to the impacting force, the impact phenomenon may be regarded as the free oscillation of a system consisting of a beam and an elastically attached load. In this regard, the displacing impact force and internal energy factors may be represented as time functions in explicit form.

The relationships obtained are valid only during the first period of impact, while the impact force exceeds zero.

For the case of complex bar frames it is recommended replacing the distributed mass by concentrated (point) masses, reducing the system to a form with plural degrees of freedom.

V. L. Biderman

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2486. Verigo, M. F., Vertical forces acting on a track when rolling stock passes along it (in Russian), *Trudi Vses. n.-i in-ta zh.-d. transp.* no. 97, 25–288, 1955; *Ref. Zh. Mekh.* no. 10, 1956, Rev. 6473.

A railway track is repeatedly loaded with various dynamic loads which change their value according to the length of the track and to the time. As a result of this, accidents to the most important constructional elements of the track, which determine the safety of movement of trains, are caused by the development of a fatigue process in the metal. The forces of interaction of the track and the rolling stock depend both on the mechanical and geometrical characteristics of the track and on the characteristics of the coaches.

The basic problem of the work is the determination of the vertical dynamic forces acting on the railway track. For its solution the theory of probability is used, a factor which is the distinguishing feature of the book. In the second chapter attention is drawn to the weight component of the vertical pressure, which is found to be a variable as a result of the redistribution of the weight of the coach along the axes as a result of the force of gravity and the unevenness of the track. The vertical components from the force of inertia of the unsprung and non-balanced masses, and also the vertical pressures of the force of the steam on the leading wheels of locomotives are examined in the third and fourth chapters.

Chapters five, six, seven, eight and nine contain a fairly detailed exposition of the dynamic interaction of the wheel and the track. The motion of a wheel having a flat spot, a smooth isolated discontinuity and a number of continuous inequalities and the dynamic action of isolated and continuous inequalities of the track are examined. The vertical pressure caused by oscillations and by the influence of the sprung structure of the coach are discussed in chapters ten and eleven.

Chapter twelve is devoted to the determination of the law of distribution of probabilities of the action of forces which make up the vertical pressure of the wheels on the rails, and to the determination of the probabilities of the appearance of various stresses in rails and ballast and on the basic area of the earth bed.

Together with rigorous methods of solving the problems, approximate ones are also indicated. Comparison of the results of the approximate calculations with experiment is given in chapter thirteen. In chapter fourteen attention is drawn to the calculation of the real conditions of operation of the track and of the rolling stock of railways.

The calculation method set out in the work makes it possible to evaluate the effect of deviations in the track characteristics and the coach portion of the rolling stock on the values of the forces of interaction of wheels and rails and on the values of the stresses occurring under the action of these forces in elements of the track.

The work contains much experimental and factual material.

Other works on the questions discussed are analyzed and criticized.

K. S. Kolesnikov

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2487. Cox, H. L., Response of missile erector towers to sudden stops or impact loads, *J. roy. aero. Soc.* **61,** 562, 694–696 (Tech. Notes), Oct. 1957.

Elasticity Theory

(See also Revs. 2408, 2456, 2474, 2479, 2512, 2514, 2525, 2532, 2536, 2546, 2551, 2557, 2578, 2580, 2589, 2624, 2875)

Book—2488. Taylor, G. I., Scientific papers, Vol. I: Mechanics of solids, New York, Cambridge University Press, 1958, x + 593 pp. $14.50.

The first volume of "The Scientific Papers of Sir Geoffrey Ingram Taylor" contains his work on the mechanics of solids. The collection of forty-one papers, arranged in chronological order, starts with the joint paper by Taylor and Griffith on "The use of soap films in solving torsion problems" published more than forty years ago in the *Proceedings of the London Institution of Mechanical Engineers* and ends with the paper on "Strains in crystalline aggregates" that Sir Geoffrey presented three years ago at the Madrid I.U.T.A.M. Colloquium on "Deformation and flow of solids". Three other volumes of his scientific papers will soon appear and will include Taylor's fundamental contributions to oceanography and turbulent motion of fluids, to meteorology, to high-speed flow and mechanics of explosions, and to miscellaneous topics in fluid dynamics. All four volumes are edited by Dr. G. K. Batchelor and are published by the Cambridge University Press.

Although the present volume covers only a part of Sir Geoffrey's multiform scientific production, it shows how influential his work and thought have been in the field of mechanics. This volume includes the celebrated papers on elasticity, plasticity, high-speed testing of materials and properties of metals, as well as Taylor's great paper on "The mechanism of plastic deformation of crystals" in which the theory of dislocations is presented and a model for the arrangement of dislocations in a crystal work-hardened by plastic strain is first proposed. As is universally recognized, Taylor's pioneer work was fundamental to the development in this field which has since occurred.

Various papers, written during World War II for British Government agencies or for governmental advisory committees, are presented to the public for the first time in this volume. Among these last papers can be found the one "On the propagation of earth waves from an explosion," the one on "The plastic wave in a wire extended by an impact load," and the joint paper with the late R. M. Davies "On the mechanical properties of cordite during impact stressing" in which a modified Hopkinson pressure-bar was used for the first time to measure stress-strain relationships of materials under dynamic loading. Also included in this volume is Taylor's celebrated 1946 James Forrest lecture to the London Institution of Civil Engineers on "The testing of materials at high rates of loading" in which theories and experiments on solids under high-speed loading are so well presented.

In reviewing part of the collected papers of one of the greatest applied mechanicists of our time, whose work has ranged over the whole domain of natural philosophy, one realizes that, in line with British tradition, for Sir Geoffrey Taylor also "Mathematics has been the servant and assistant, not the master. His guiding star in science has been natural philosophy."

E. Volterra, USA

2489. Zizicas, G. A., Reduction of three-dimensional stress distributions to two-dimensional analysis by superposition, *J. appl. Mech.* **24**, 3, 478–480, Sept. 1957.

2490. Abramyan, V. L., One case of a plane problem of the theory of elasticity for a rectangle (in Russian), *Dokladi Akad. Nauk ArmSSR* **21**, 5, 193–198, 1955; *Ref. Zb. Mekb.* no. 12, 1956, Rev. 8451.

The plane problem of the theory of elasticity for a rectangle is examined in the conditions of edges loaded symmetrically relative to its axis. The function of stress is sought in the form of

$$\varphi(x,y) = \sum_{k=1}^{\infty} \cos \alpha_k x \,\{A_k^{(1)}\mathrm{ch}\,\alpha_k y + B_k^{(1)}\mathrm{sh}\,\alpha_k y + \alpha_k y(C_k^{(1)}\mathrm{ch}\,\alpha_k y + D_k^{(1)}\mathrm{sh}\,\alpha_k y)\} + \sum_{k=1}^{\infty} \cos \beta_k y\,\{A_k^{(2)}\mathrm{ch}\,\beta_k x + B_k^{(2)}\mathrm{sh}\,\beta_k x + \beta_k x(C_k^{(2)}\mathrm{ch}\,\beta_k x + D_k^{(2)}\mathrm{sh}\,\beta_k x)\}$$

$$\left(\alpha_k = \frac{(2k-1)\pi}{2l},\ \beta_k = \frac{(2k-1)\pi}{2b}\right)$$

Here $2l$ = length of the rectangle, $2b$ its height, while coefficients $A_k^{(s)}, B_k^{(s)}, C_k^{(s)}, D_k^{(s)}$ $(s = 1,2)$ depend on their determination from the boundary conditions; the last are also described in the Fourier series.

From the conditions of symmetry it follows that $B_k^{(1)} = C_k^{(1)} = B_k^{(2)} = C_k^{(2)} = 0$. After the exclusion of constants $A_k^{(1)}, A_k^{(2)}$ for the remaining unknown quantities $D_k^{(1)}, D_k^{(2)}$ an infinite system of equations is obtained; the regularity of the obtained system is demonstrated.

V. K. Prokopov

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2491. Baslavskii, I. A., Solution of a plane problem of the theory of elasticity for a bar of variable section by the method of successive approximations (in Russian), 13-ya nauch. Konferentsiya Leningr. inzh.-stroit. in-ta, Leningrad, 1955, 206–208; *Ref. Zb. Mekb.* no. 12, 1956, Rev. 8450.

An examination is made of a strip, bounded by curves $y_{1,2} = \pm b[1 + \lambda\omega(x)]$ which, substituted by variables $\xi = x$, $\eta = y[1 + \lambda\omega(x)]^{-1}$, is converted into strip $\eta = \pm b$, where λ = the small parameter. Stresses and mass forces in these variables are presented in the form of a series

$$\sigma_x(x,y) = \sum_{n=0}^{\infty} \lambda^n \sigma_{nx}(\xi,\eta), \text{ etc.}$$

By interpolating these expressions in the equation of equilibrium, in the equation of joint deformation and boundary conditions, and at the same time assuming that the sum of coefficients with λ with identical ratios equals zero, author obtained a system of equations from which, successively, values are arrived at for $\sigma_{nx}, \sigma_{ny}, \tau_{nxy}$.

I. A. Prusov

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2492. Khristoforov, V. V., The use of Green functions in static two-dimensional problems of the theory of elasticity (in Russian), *Trudi in-ta matem. i mekhan. Akad. Nauk UzSSR* no. 15, 131–141, 1955; *Ref. Zb. Mekb.* no. 10, 1956, Rev. 6829.

Using the notation

$$\theta \equiv \frac{\partial u}{\partial x} + \frac{\partial y}{\partial y}, \quad \omega \equiv \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

where u and v are the components of the displacement vector, author obtains for the case of the first problem in the two-dimensional of elasticity (the displacements u_s and v_s on the contour L of the region S being stated), when the Green function $\Gamma \cdot G - 1nr$ of the

Dirichlet problem is known, the following integral equations:

$$\theta(t) = \theta^* - \frac{\lambda + \mu}{\pi(5\lambda + 9\mu)} \int_L \left[(x_s - x_t) \frac{\partial}{\partial x_t} \left(\frac{\partial \Gamma}{\partial n} \right) + (y_s - y_t) \frac{\partial}{\partial y_t} \left(\frac{\partial \Gamma}{\partial n} \right) \right] \theta(s) ds$$

$$\omega(t) = \omega^* - \frac{\lambda + \mu}{\pi(\lambda - 3\mu)} \int_L \left[(x_s - x_t) \frac{\partial}{\partial x_t} \left(\frac{\partial \Gamma}{\partial n} \right) + (y_s - y_t) \frac{\partial}{\partial y_t} \left(\frac{\partial \Gamma}{\partial n} \right) \right] \omega (S)\, ds$$

in which

$$\theta^* \equiv \frac{4\mu}{5\lambda + 9\mu} \lim_{q \to t} \theta^* (q), \ \omega^* \equiv - \frac{\lambda + 3\mu}{4(\lambda + 2\mu)} \lim_{q \to t} \omega^* (q)$$

$$u^* (q) = - \frac{1}{2\pi} \int_L \frac{\partial \Gamma}{\partial n} u_s ds, \quad v^* (q) = - \frac{1}{2\pi} \int_L \frac{\partial \Gamma}{\partial n} v_s ds$$

In the case of the second problem of the plane theory of elasticity (when the stresses on the contour are stated), when the Green function of the Neumann problem in θ and ω is known, the author likewise obtains integral equations in which integration is already performed for the region S. A. K. Rukhadze

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2493. Rosenberg, L. A., Direct variational methods in the complex problem of the theory of elasticity (in Russian), *Trudi in-ta mat. i mekh. Akad. Nauk UzSSR* no. 13, 71–91, 1954; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7665.

An investigation of the equilibrium of an elastic semi-space with specific displacements on one part of the limiting interface, and surface forms on the remaining part. The direct method of mathematical physics according to the Betti equation is applied. Functional analysis is used to demonstrate the fundamental propositions on the convergence of the solutions in the mean, and of the first derivatives. The application of the method to contact problems is illustrated. I. S. Arzhanykh

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2494. Atsumi, A., Stress concentrations in a strip under tension and containing two pairs of semicircular notches placed on the edges symmetrically, *J. appl. Mech.* **24,** 4, 565–573, Dec. 1957.

Considering the strip to be unnotched and calculating the radial and tangential stresses along the circumference of each notch resulting from the tensile force and then applying to the edge of each notch stresses equal to the negative of the calculated stresses, the problem is reduced to an unnotched strip under tension with additional concentrated longitudinal and transverse forces applied at points corresponding to the center of each notch. The stress functions are found using a method first suggested by R. C. Knight [*Quart. J. Math.* **5**, 255–268, 1934].

Numerical results are given for two examples and are compared with theoretical investigations of the corresponding problem for a strip containing only one pair of notches and with photoelastic tests [AMR **5** (1952), Rev. 3044; **6** (1953), Rev. 1527].

Paper is a valuable contribution to the question of how much the values of stress concentration in a strip decrease as the notches increase in number. E. Seydel, Germany

2495. Miyamoto, H., On the problem of elasticity theory for an infinite region containing three spherical cavities, Proc. Sixth Japan nat. Congr. appl. Mech., Univ. of Kyoto, Japan, Oct. 1956, 27–30.

Author finds the state of stress in an elastic body under the action of uniform tension at infinity, containing three spherical cavities the centers of which are on a line. Calculations are given when the three are of equal size, and the results are compared with the case of two cavities previously solved by the author [Proc. Fifth Japan nat. Cong. appl. Mech. 1955, 125–129].

E. Saibel, USA

2496. Verma, G. R., Stresses in a circular cylinder and in a paraboloid of revolution due to shearing forces produced by circular rings of the curved surface, *Indian J. theor. Phys.* **4,** 4, 93–98, Dec. 1956.

Paper deals with two problems of torsion closely related to each other. The first problem treated is the torsion of an isotropic circular cylinder. The torsional forces are produced with the help of two closely fitting rings, arranged at equal distances from the plane ends on the curved surface of the cylinder. As second problem the torsion of an isotropic paraboloid of revolution is dealt with, the end of which terminates on a confocal paraboloid. The shearing forces are applied by a close-fitting ring on the curved surface. Both problems are treated in an elegant way using Dirac's delta function. As result of the investigation, the components of stresses are expressed by simple series.

P. Csonka, Hungary

2497. Csonka, P., Torsion of box beams (in Hungarian), *Magyar Tud. Akad. Oszt. Közl.* **18,** 1/4, 201–211, 1956.

2498. Sonnemann, G., and Davis, D. M., Stresses in long thick-walled cylinders caused by pressure and temperature, ASME Ann. Meet., New York, N. Y., Dec. 1957. Pap. 57-A-256, 25 pp.

Paper presents known analysis of stress in infinitely long thick-walled tubes made of homogeneous isotropic perfectly elastic material, and submitted to pressure and logarithmic temperature distribution with axial symmetry. Results are presented in graphical nondimensional form. In addition, a new problem is solved: stresses caused in such tubes by nuclear internal radiation. Known simplification of slab temperature solution is not used; integration of heat-conduction equation is performed without introducing it, and exact results are evaluated for the stresses for these boundary conditions: (a) equal temperature on both surfaces, (b) inside surface insulated, (c) outside surface insulated, (d) bulk temperatures of coolant in both inside and outside channels are equal, heat-transfer coefficient at inner surface is almost equal to outer heat-transfer coefficient multiplied by diameter ratio.

Results for nuclear radiation are rather unexpected: maximum tensile stresses occur always at same radial position as lowest temperature; maximum compressive stresses occur often at hottest point, maximum tangential stresses depend upon boundary conditions. H. Fernandez Long, Argentina

2499. Smith, R. C. T., Tension of an infinite plate cut along a circular arc, *J. Math. Phys.* **36,** 3, 223–233, Oct. 1957.

Author investigates a case with same title as Muskhelishvili's text book, § 124a, the cut along the unit circle from $-\beta$ to 0 to $+\beta$ now being lubricated by a thin layer transmitting normal stresses but no tangential shears. Analogous to Muskhelishvili's developments, additional stresses must show characteristic behaviour along the cut and passing to infinity, and convenient singularities at end points of the cut. Finally, problem reduces to three functions whose complex constants are to be found by an equation system. Formulas are given for stresses along the cut edges and dis-

placements there, and the change of strain energy due to the crack. Numerical values are given for relative displacements and change of energy for cuts with $\beta = 0 - 30°$ and 45° direction of undisturbed stress to symmetry axis. W. Mudrak, Austria

2500. Sharma, B., Thermal stresses in infinite elastic disks, *J. appl. Mech.* **23**, 4, 527-531, Dec. 1956.

Author modifies Beltrami's equations to include thermal effects. He uses particular solutions to find the stress in a disk of finite thickness and infinite radius, subject to an axisymmetric temperature distribution. D. R. Bland, England

2501. Zaitsev, G. P., Questions of the elasticity constant and the bulk modulus: calculation of large deformations (in Russian), *Fiz. Metallov i Metallovedeniye* **1**, 2, 193-205, 1955; *Ref. Zb. Mekb.* no. 11, 1956, Rev. 7964.

A new nondimensional elasticity constant is proposed, in the forn $z = \frac{1}{3} E\chi$, where E = modulus of elasticity, χ ratio of the relative elastic volumetric change to the hydrostatic pressure. Expressions are set up for the bulk modulus, applying to any values of the elastic, plastic, and elastic-plastic deformations. These have been verified experimentally for a number of different materials. The conception is introduced of the three moduli of elasticity; in particular, it is found that, in the case of large deformations with a zero value of the spherical tensor, an elastic change in volume takes place. For instance, in pure shear and torsion, the volume is increased, which agrees with the conclusions of other authors.

V. G. Osinov

Courtesy Referativnyi Zburnal, USSR

Translation, courtesy Ministry of Supply, England

Experimental Stress Analysis

(See Revs. 2546, 2616)

Rods, Beams, Cables, Machine Elements

(See also Revs. 2418, 2425, 2452, 2454, 2455, 2458, 2476, 2482, 2483, 2484, 2488, 2491, 2497, 2527, 2528, 2533, 2542, 2570, 2579, 2603, 2628)

2502. Maunder, L., The bending of pretwisted thin-walled beams of symmetric star-shaped cross sections, *J. appl. Mech.* **25**, 1, 67-74, Mar. 1958.

Analysis is made of the bending deflections of a pretwisted thin-walled beam of doubly symmetric cruciform cross section. Energy method is used which takes into account the transverse distortion of the fins resulting from their helical shapes. This is a refinement over previous analyses and predicts larger deflections. Theory shows good agreement with test results.

G. W. Housner, USA

2503. Holmes, A. M. C., Analysis of helical beams under symmetrical loading, *Proc. Amer. soc. civ. Engrs.* **83**, ST 6 (*J. Struct. Div.*), Pap. 1437, 38 pp., Nov. 1957.

Contrary to a previously given rigorous analysis of a helical beam [AMR **4**, Rev. 1495] only symmetrical loading is considered in order to arrive at results valuable to the design engineer. The solution, based upon the theory for slightly curved, slender beams, suggests that thin helical slabs are structurally more efficient than helical beams with deep narrow cross sections.

J. F. Besseling, Holland

2504. Vaughan, W. L., Simplified graphical integration for solution of beam problems, *Prod. Engng.* (Design Digest Issue) **28**, 15, F26-F27, Oct. 1957.

2505. McCallum, J., Bending moments in bracketed beams, *Quart. Trans. Instn. nav. Arch.* **99**, 2, 173-203, Apr. 1957.

Paper deals with results obtained from the "Glengarnock" experiments, carried out by Lloyds Register of Shipping for the British Shipbuilding Research Assn., and describes a method of calculation of bending moments, stresses, and deflections of panels of plating with bracketed stiffeners, as used in shipbuilding. Relationship between bending moment factor and rigidity and the optimum size of brackets are analyzed. End structure rigidity, position of neutral axis and effective width, twisting and tilting, riveting and equivalence of stress in riveted and welded beams are investigated. Approximations are presented for the inertia factor and end rigidity. In appendices, general solutions for the relationship between stiffness and bending moment factor, end structure rigidity including bulkhead stiffeners, and connections of heavy members are discussed; a typical deep tank bulkhead is described; and some of the Glengarnock test results are analyzed. P. W. Abeles, England

2506. Crandall, S. H., The Timoshenko beam on an elastic foundation, Proc. Third Midwestern Conf. on Solid Mech., Univ. of Mich., Apr. 1957, 146-159.

The effect of the shear is included in the differential equation of motion of the beam on elastic foundations. The foundation is assumed, as usually, to exert a restoring force proportional to the local deflection.

Critical wave velocities are obtained for a transverse disturbing force varying harmonically and for a constant moving load.

W. Ornstein, USA

2507. Yelisuisky, G. G., The calculation of continuous beams of constant rigidity without simultaneous solution of a system of equations (in Azerbeidjani), *Trudî Azerb. industr. in-ta* no. 10, 50-62, 1955; *Ref. Zb. Mekb.* no. 11, 1956, Rev. 7859.

A method is presented for calculating continuous beams (with from 2 to 5 spans) on rigid supports, leading to the substitution of available data in previously derived formulas, suitable for any static load. The calculation consists in setting up and solving, for each supporting moment, a single, fundamental equation with one unknown, the external load being represented by corresponding fictitious supporting reactions. Tables have been constructed for calculating beams with freely supported ends and with constrained end supports. N. L. Kuzmin

Courtesy Referativnyi Zburnal, USSR

Translation, courtesy Ministry of Supply, England

2508. Kushelev, N. Yu., Stresses in the bracing of a compound beam, loaded with a concentrated force in the span (in Russian), *Trudî Leningr. politekhn. in-ta* no. 178, 188-199, 1955; *Ref. Zb. Mekb.* no. 12, 1956, Rev. 8600.

A cantilever girder is examined, consisting of two rectangular braces, connected along their whole length by an uninterruptedly distributed elastic bond. The load, in the shape of a concentrated force, is applied at some arbitrary distance from the free end. An investigation is made of the rule of distribution along its length of the tangential stresses in the bonds on the assumption that the characteristic of the bonds shall be linear.

It was found that the evaluation of the flexibility has a practical meaning, especially for wooden beams. For steel beams, the proposed solution has a practical significance when the load is dis-

tributed close to the clamped end and when the modulus of elasticity of the bonds is large. S. V. Boyarshinov

Courtesy Referativnyi Zburnal, USSR
Translation, courtesy Ministry of Supply, England

2509. Klimovich, D. L., A method of graphical analysis for determining the stresses in the constrained torsion of thin-walled bars (in Russian), *Nauch. rab. stud. Leningr. inzb.-stroit. in-ta* no. 2, 19-24, 1955; *Ref. Zb. Mekb.* no. 11, 1956, Rev. 7856.

The solution is founded on applying the existing analogy between the differential equation of the torsional angles in gradual torsion and the differential equation of bending deflections of a bar bent in tension. The procedure of graphical analysis ultimately reduces to the construction of two rope polygons and subtraction of their ordinates after multiplying them by a specific coefficient. E. I. Silkin

Courtesy Referativnyi Zburnal, USSR
Translation, courtesy Ministry of Supply, England

2510. Yegupov, V. K., An investigation into the resistance of flat decks with consideration of beam torsion (in Uzbek), *Doklady Akad. Nauk UzSSR.* no. 6, 31-36, 1955; *Ref. Zb. Mekb.* no. 11, 1956, Rev. 7854.

The problem of the resistance of a deck can be resolved into a series of partial problems relating to the resistance of isolated beams on elastically sinking and elastically rotating supports. The resolution is performed by transforming the matrix equation for the stability of decks or ceilings; this problem can be solved by elementary means.

The method of resolution leads to a practically acceptable solution only if the problem of the stability of the deck or ceiling is divided into two problems of beam resistance. This corresponds to a limitation in the number of types of ceiling, even for the simplest case of unidirectional beam compression. In the case of bidirectional compression, the number of types is further reduced, while consideration of the torsional rigidity leaves only one type, which, regrettably, the author does not specify.

Stability analysis by resolution is inapplicable practically to decks or ceilings of a general type. A. A. Kurdyumov

Courtesy Referativnyi Zburnal, USSR
Translation, courtesy Ministry of Supply, England

2511. Soule, J. W., Tensor flexibility analysis of closed-loop piping systems, *J. appl. Mech.* **25,** 1, 11-16, Mar. 1958.

Author adopts a topological point of view in analyzing rigid elastic structures. He explains how the geometrical concepts of spatial positions can be separated from the physical concepts of flexibilities by constructing a connection matrix $\overline{C}$. Especially valuable is the comparison of electrical concepts (such as a battery voltage *e*) with structural concepts (such as a mismatch of deflections). The process of setting up the equation of state of an elastic structure follows step by step those of an analogous electrical network, with the difference that each scalar quantity (current, voltage, etc.) associated with a coil is replaced by a vector quantity with six components (forces, displacements, etc.) associated with a pipe or beam. All steps are discussed in detail.

G. Kron, USA

2512. Das, S. C., On the stresses in a composite truncated cone due to shearing stresses on the curved surface, *Indian J. theor. Phys.* **4,** 4, 89-92, Dec. 1956.

The torsion of a truncated cone composed of two different materials is dealt with. The cones are coaxial, the uniform shearing stresses are applied on the outer curved surface. To solve the problem, conical harmonic functions are introduced. The displacements are expressed by infinite convergent series. The deduced formulas are relatively simple.

P. Csonka, Hungary

Plates, Disks, Shells, Membranes

(See also Revs. 2460, 2461, 2462, 2463, 2464, 2488, 2498, 2500, 2532, 2534, 2535, 2536, 2537, 2538, 2539, 2545, 2557, 2578, 2588)

Book—2513. Lekhnitsky, S. G., Anisotropic plates (in Russian), Moscow, Gosud. Izdat. Tekh.-Teor. Lit., 1957, 463 pp. 16.55 rubles.

This is the second edition of a book originally published in 1944 and reprinted in 1947 [AMR **2** (1950), Rev. 722]. The current edition covers the same basic material treated in the earlier one but is much more timely and presents material published as late as 1957. The present edition appears to be a fairly complete summary of all existing knowledge in the field of anisotropic plate theory. Much of the material included is original work due to the author himself and has previously been published in various Russian-language periodicals.

Chapter I presents the fundamental equations of the theory of the elastic anisotropic bodies. Only the case of linear strain-displacement relations is considered. Chapter II presents a number of plane stress and plane strain problems for anisotropic bodies, including the torsion problem, and also several problems pertaining to circular disks loaded by various distributions of normal forces around the circumference. Chapter III presents a collection of plane stress problems involving the bending of straight and curved beams. Various conditions of end support as well as cantilever beams subject to pure bending, uniform lateral load, or hydrostatic loadings are considered. The stresses arising in wedges subject to arbitrary forces applied at the tip are also treated. Lastly, the pure bending as well as uniform radial loading of initially circular beams is considered. Thin-walled cylinders and laminated cylindrical pressure vessels subject to internal or external pressure are also treated.

Chapter IV discusses the distribution of stresses in infinite elastic media and generalizes the Boussinesq problem to the case of anisotropic bodies. Chapter V treats the stresses in elliptical and circular plates subject to a variety of loadings acting in the plane of the plate and distributed around the boundary. Chapter VI considers the stresses in various shaped plates with small elliptical and circular openings, the loadings being in the plane of the plate. Much of the work presented in this chapter is due to Savin [AMR **10** (1957), Rev. 1025]. Problems treated include infinite plates with elliptical holes in which the boundaries of the holes are loaded by either uniform normal pressure or uniformly distributed shear forces. Certain cases involving finite plates are also considered. Chapter VII is an entirely new chapter that does not appear in the previous edition. It treats various approximate methods of determining stresses in plates with only slight anisotropy. The method of perturbations is used extensively in these treatments. Chapter VIII discusses various approximate methods of determining stresses in anisotropic plates with various shaped openings. Only plane stress problems are treated. Problems concerning infinite plates having triangular or rectangular holes with rounded corners, the infinite plate being subject either to uniaxial tension or pure bending, are discussed in detail.

Chapter IX presents a general theory of bending of thin anisotropic plates. Here for the first time in the book nonlinear strain displacement relations are introduced and the generalized von Karman equations for anisotropic plates are presented. Chapter X presents solutions of a variety of problems involving thin plates subject to normal loadings. Rectangular, circular, elliptical and triangular plates with various edge conditions and subject to either concentrated or uniformly distributed loads are considered. The case of cylindrical bending of long rectangular plates is also treated in detail. Chapter XI treats bending of plates loaded only by forces applied at the boundaries of the plates. Problems such as the bending of rectangular plates by moments distributed along

the edges (the plates having circular or elliptical cut-outs at their centers) are treated. Chapter XII considers the transverse vibrations of anisotropic rectangular, elliptical, and triangular plates.

Chapter XIII presents the basic theory of elastic stability of anisotropic plates, but only the linearized theory is discussed. The last three chapters of the book present a large number of special cases of plate buckling problems. Chapter XIV considers the buckling of plates loaded by distributed forces lying in the plane of the plate. Chapter XV discusses the solutions when concentrated forces are acting, and in Chapter XVI the buckling of rib-stiffened plates is investigated.

The current edition summarizes the present state of knowledge in the specialized area of anisotropic plate theory. The book is well-referenced, and the author demonstrates a full awareness of Western contributions to this field.

W. A. Nash, USA

2514. Bassali, W. A., and Dawoud, R. H., Bending of an elastically restrained circular plate under normal loading over a sector, *J. appl. Mech.* **25**, 1, 37–46, Mar. 1958.

Authors expand the given load on the sector into a Fourier series and use the complex variable theory to solve the biharmonic equation for the thin-plate deflection. Expressions for deflections, moments, shear forces, and reactions on the boundary are obtained for the boundary condition in which the ratio of the boundary normal moment to the boundary tangential moment is a constant. Curves of deflections and moments are shown for various values of this constant ratio of moments on the boundary.

B. E. Gatewood, USA

2515. Frasier, J. T., and Rongved, L., Force in the plane of two joined semi-infinite plates, *J. appl. Mech.* **24**, 4, 582–584, Dec. 1957.

Two semi-infinite plates of different materials are joined along a plane interface. Parallel to middle plane a force acts at a point interior to one of the plates.

Paper presents formulas in closed form for stress distribution in the two plates. Derivation of results is omitted.

H. Parkus, Austria

2516. Tekinalp, B., Elastic-plastic bending of a built-in circular plate under a uniformly distributed load, *J. Mech. Phys. Solids* **5**, 2, 135–142, Mar. 1957.

Paper contains an analysis of the bending moments and deflections of a uniformly loaded and built-in circular plate that is made of an incompressible elastic-plastic material obeying Tresca's yield condition and the associated flow rule. The analysis is simplified by assuming that any plate element is either entirely elastic or entirely plastic. This assumption is practically fulfilled for a sandwich plate; for a solid plate it represents a first approximation to the actual diagram of bending moment versus curvature.

From author's summary by Y.-H. Pao, USA

2517. Dorosh, N. A., The application of a concentrated load to the reinforcing ring of a circular cutout in an infinite plate (in Russian), Byul. nauk. stud. Konferentsii, Pt. II, Lvov, Univ. Publishers, 1955, 102–105; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7687.

The case is examined of a plate of infinite extent, having a circular cutout reinforced along its edge by a thin circular ring, to the free edge of which a concentrated force is applied, acting at an angle to the normal.

I. A. Prusov

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2518. Jones, P. D., Small deflection theory of flat plates using complex variables. I. Fundamental equations in complex form, *Aero. Res. Lab. Rep., Melbourne* SM-252, 21 pp. + 4 figs., June 1957.

In the present paper the equations of the small-deflection theory of thin elastic plates are rewritten in a form suitable for using complex variable methods. The plates considered are such that they can be mapped onto the unit circle by polynomial transformation functions, and the loading cases considered consist of continuously differentiable pressure distributions, point loads, line loads, and regional loads.

Y.-H. Pao, USA

2519. Fel'dman, M. R., The evaluation of flexible plates (in Russian), *Dop. Akad. Nauk URSR* no. 5, 451–457, 1955; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8467.

An examination is made of rectangular absolutely flexible plates (membranes) which are under the influence of evenly distributed loads; points on the contour are taken to be not subject to displacement. The method of energetics is made use of; each of the transferred points u, v, w of the median plane is expressed as a trigonometrical series. For the calculation of the square of the membranes three terms of each series are withheld. Diagrams are given for the deflection and stresses in various points of the membranes. Results are compared with the solution in the first approximation according to Fepplyu [A. Feppl, L. Feppl, "Stresses and deformation," Gosteizdat., 1933, 254–258].

For membranes with sides in the ratio $b/a = 1.5$, 2, 3, and 5, the problem is solved by withholding one term of the series. However when elongated membranes were examined ($b/a > 2$), such an approximation could not be justified.

A. S. Vol'mir

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2520. Zvereva, K. D., On the bending of a thin plate of varying thickness (in Russian), *Trudi Mosk. energ. in-ta* no. 17, 47–53, 1955; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8475.

An approximate method is submitted for the solution of the problems of equilibrium of a rectangular plate of variable thickness, the stiffness of which changes in accordance with the linear rule, leading to the substitution of the equations of particular derivatives

$$\Delta w = \frac{M}{D}, \ \Delta M = P$$

by the system of ordinary linear differential equations.

V. K. Prokopov

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2521. Lavrinenko, P. P., Stress analysis of the eye of a paddle-wheel arm in a river steamer (in Russian), *Izv. Kievsk. politekhn. in-ta* **18**, 53–70, 1955; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7901.

With some simplifying assumptions, author reduces the title problem to the calculation of the stresses in a steel plate with a rounded end and a circular hole (eye) therein, associated with a roller acted upon by coplanar forces.

The contact (bearing) stresses between the plate and the roller are determined by the theory of I. Ya. Staerman ["The contact problems of the theory of elasticity," Moscow, 1949].

By usual means, and in consideration of the smallness of the aperture, the problem is reduced to the conventional case of a circular ring specifically loaded on both edges.

I. G. Aramanovich

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2522. Ambartsumian, S. A., The calculation of laminated anisotropic shells (in Russian), *Izv. Akad. Nauk ArmSSR, Ser. fiz.-matem., estestv. i tekhn. nauk* **6**, 3, 15–35, 1953; *Ref. Zh. Mekh.* no. 10, 1956, Rev. 6856.

A general theory is developed for the elastic equilibrium of a thin shell built up of an arbitrary number of homogeneous anisotropic laminae, working in conjunction without slipping. It is assumed that each lamina is of constant thickness, follows the generalized Hooke's law, and experiences small deformations; the thickness ratio of the different laminae is assumed to be arbitrary, as well as the ratio of the corresponding moduli of elasticity (the structure of the laminae being generally asymmetrical to the median surface).

Assuming the hypothesis of rectilinear nominals to apply to the case of a laminated shell, author expresses the deformation components of each lamina by the projection of the displacements of one of the external surfaces, and thence establishes the expressions for the component stresses in each lamina. The area stresses in the fundamental sections in the general case are reduced to tangential (longitudinal) and transverse forces, and to the bending and torsional moments, these forces and moments satisfying the equations of equilibrium, which have the same form as in the case of a homogeneous shell.

The following cases are examined in more detail: (1) A shell of rotation, experiencing axially symmetrical deformations; (2) a flat shell; and (3) a flat sheet (flat plate). In the case of the shell of rotation, two auxiliary functions $U(s)$ and $V(s)$ are introduced for the arc of the meridian s, expressing the components of deformation, forces and moments; applying the condition of simultaneity of the deformations, the author obtains a system of two second-order differential equations for the functions U and V.

In the case of the flat shell (envelope), after certain simplifications, a system of three differential equations for the projections of the displacements U, V and W is obtained. Further, a function ϕ is introduced which expresses the tangential forces (by expressions resembling those for the two-dimensional case); for this function ϕ, and for the displacements W in the direction of the normal to the median plane, a system is obtained of two conjugated differential equations of the fourth order, with coefficients depending on the elasticity constants of the laminae. This system is equivalent to a single equation of the eighth order, satisfied by a further function Φ, analogous to a similar equation for the homogeneous shell.

In this particular case, the equations of the flat envelope become transformed with the equation of the flat plate. Differing from the case of a panel with symmetrically arranged laminae, the stress function ϕ and bending deflection w of the plate of asymmetrical construction satisfy a system of two simultaneous equations. This indicates that, in the bending of such a plate by external flexural forces and moments, internal longitudinal strains are apparent, acting in the median plane; and, conversely, in the case of deformation by external forces acting in the median plane, both internal bending and internal shearing strains become apparent.

The example is examined of the bending of a square plate with freely-supported edges, consisting of two isotropic laminae with a thickness ratio of 1 : 2. The solution is obtained by the method of double trigommetric series, and developed to give numerical results.

S. G. Lekhnitskii

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2523. Wittrick, W. H., Edge stresses in thin shells of revolution, *Aero. Res. Lab. Rep., Melbourne* SM-253, 31 pp. + 2 figs., June 1957.

In solving for the stresses in thin shells, the substitution of the membrane-theory equations for the full shell-theory equations implies that all forces other than the internal tensions and shear forces parallel to the surface are neglected. Although permissible over the major portion of the shell, this simplification fails at the boundaries when bending stresses can be of the same order of magnitude as the membrane stresses.

This paper reports on a method of satisfying all the edge conditions by superimposing on the membrane stresses an additional stress system which is governed by the full shell equations. It is known that, if the shell is thin, these additional stresses decay rapidly as the distance from the edge increases. This fact is exploited to obtain an approximate solution of the full shell equations for the additional stress system.

Y.-H. Pao, USA

2524. Malyutin, I. S., One contact problem on cylindrical shells (in Russian), *Prikl. Mat. Mekh.* **20**, 5, 665–666, Sept.–Oct. 1956.

Problem is that of two well-fitted circular cylinders subjected to an external concentrated radial pressure P. The cylinders are thin and infinite, the contact is frictionless. Contact pressure $q(x)$ between cylinders may be $\gtrless 0$.

The radial deflections of both cylinders are equated to yield an integral equation for $q(x)$. Solution for $q(x)$ is obtained by using a double Laplace transform.

Contact pressure $q(x)$ is found to be made of a concentrated radial pressure and of a distributed pressure (alternately > 0 and < 0) dying off exponentially.

H. Beguin, USA

2525. Kochanski, S. L., and Argyris, J. H., Some effects of kinetic heating on the stiffness of thin wings, *Aircr. Engng.* **24,** 344, 310–318, Oct. 1957.

A preliminary analysis is presented of the effects of kinetic heating at supersonic speeds on the torsional and flexural stiffness of thin solid wings. The heat-transfer and heat-conduction aspects are discussed first, with numerical conclusions for certain assumed flight paths. The resulting thermal stresses are derived and numerical results are presented. The influence of thermal stresses on torsional stiffness for both small and large twisting displacements is discussed. The influence of thermal stresses on bending stiffness for small bending curvature is shown. This latter influence derives from the fact that the anti-elastic curvature is altered by the thermal stress pattern. Finally, there is a discussion of means for alleviating the effects of kinetic heating in thin wings of the type considered in the paper.

R. L. Bisplinghoff, USA

2526. Fulop, W., Some further results on the rubber membrane theory and Laplace's equation, *J. sci. Instrum.* **34,** 11, 453–454, Nov. 1957.

In a previous paper [AMR **8** (1955), Rev. 2689], author has shown that the usual assumption that the rubber membrane analogue of Laplace's equation is valid only to a first-order approximation and is less applicable if the inclination of the membrane to the xy-rest-plane is too large is, in fact, too restrictive. Rather, revised theory, stemming from experimental rubber-membrane determination of the electrostatic capacitance of two circular coaxial cylinders, evidences that Laplace's equation follows for all inclinations to the rest-plane subject to postulation that the "component tension T_0 in the xy-plane remains constant"; and, further, that the deflection normal to the xy-plane is in accord with Hooke's law. Further results, stemming from membrane determination of the capacitance of two circular eccentrically-located cylinders, substantiate this revised theory and manifest the considerable departure of results obtained from those accorded by previous theory. The usefulness of certain desirable theoretical

investigations of an idealized two-dimensional lattice, as suggested by the results of the revised theory, are remarked.

T. J. Higgins, USA

2527. Zyczkowski, M., Limit state of nonhomogeneous rotating disks (in Polish), *Rozprawy Inz.* **5**, 1, 49–96, 1957.

Problems concerning the determination of the limit angular velocity and the corresponding stresses in a rotating nonhomogeneous circular disk are considered. It is assumed that the moduli of nonhomogeneity (plastic or mass nonhomogeneity, etc.) are functions of radius. The computation of the limit state of a disk of variable thickness is reduced to that of a disk of constant thickness but characterized by material nonhomogeneity of a suitable type.

The yield conditions of Coulomb-Tresca and Huber-Mises-Hencky are used. The basic system of equations splits up into two groups. One of them enables the determination of the field of internal forces and the limit angular velocity. The other group of equations concerns deformations of the rotating disk.

For the Coulomb-Tresca condition, if $0 \leq \sigma_r \leq \sigma_\varphi$, limit angular velocity of nonhomogeneous disk of radius R is

$$\omega = \left(\frac{gQ_0}{R^2\gamma_0} \cdot \frac{\int_0^1 K(\rho)d\rho}{\int_0^1 K(\rho)\rho^2 d\rho} \right)^{1/2} \qquad [1]$$

where $K(\rho) = Q(\rho)/Q_0$ is a function of plastic nonhomogeneity, $K(\rho) = \gamma(\rho)/\gamma_0 \rho = \frac{n}{R}$ and g denotes the gravity acceleration, Q_0 and γ_0 the yield point and the specific weight of the material for $\rho = 0$, respectively.

Conditions to be satisfied by the nonhomogeneity function, if the limit angular velocity is calculated according the equation [1], are discussed.

The plastic nonhomogeneity function is sought using the Huber-Mises-Hencky yield condition and assuming a stress field, conformal to the boundary conditions and the angular speed.

Examples of nonhomogeneity functions are given for stresses in the form of polynomials. A method for finding (for stress fields of a definite type) a nonhomogeneity function most nearly approaching the real plastic nonhomogeneity is discussed.

The limit angular velocity of the disk

$$\omega = 1.805 \sqrt{\frac{gQ}{R^2\gamma}} \qquad [2]$$

for which the mean deviation from homogeneity of the nonhomogeneity function is 0.61%.

The Hencky-Ilyushin relations are used for the determination of deformations.

A. Sawczuk, Poland

Buckling Problems

(See also Rev. 2528)

2528. Leaf, G. A. V., A property of a buckled elastic rod, *Brit. J. appl. Phys.* **9**, 2, 71–72, Feb. 1958.

One shape assumed by a perfectly elastic rod buckled by appropriate forces and couples at its ends is studied. It is shown that the ratio of maximum height to maximum width is independent of the material of the rod. Also, this ratio is linearly related to the distance between the ends of the rod expressed as a fraction of its length, provided the fraction is roughly less than one half.

G. G. Meyerhof, Canada

2529. Clark, M. E., Sidebottom, O. M., and Shreeves, R. W., Inelastic analysis of eccentrically-loaded columns, *Proc. Amer. Soc. civ. Engrs.* **83**, EM 4 (*J. Engng. Mech. Div.*), Pap. 1418, 34 pp., Oct. 1957.

Semigraphical procedures for determining collapse load on eccentrically loaded columns assume that plane sections remain plane, that stress-strain diagram consists of two straight lines, and that deflected curve is circular or sinusoidal. Predicted loads compare favorably with experimental results of 14 columns of steel and aluminum with rectangular and T-shaped cross sections. Tests indicated collapse load to be time-dependent.

F. J. McCormick, USA

2530. Lubkin, S., Determination of buckling criteria by minimization of total energy, New York Univ., Inst. Math. Sci., AFOSR TN-57-579, 60 pp., July 1957.

Author derived the buckling criteria by using the feature that buckling occurs when small perturbations in the displacements corresponding to the unbuckled state result in a decrease in the total energy. For calculating the strain energy in the system it is assumed that the elastic constants satisfying Hooke's Law remain invariant, there being no restriction on the magnitude of strains or displacements. Problems discussed are those of (1) a rectangular column under compression, (2) a hollow cylinder subject to uniform pressure on the outer surface, and (3) a hollow sphere under uniform external pressure. In all cases, conditions of symmetric deformation were assumed.

A. M. Sen Gupta, India

2531. Au, T., Ultimate strength design charts for columns controlled by tension, *J. Amer. Concr. Inst.* **29**, 6, 471–480, Dec. 1957.

2532. Shapovalov, L. A., Influence of internal pressure on critical torsional stress for infinitely long cylindrical cylinders (in Russian), *Prikl. Mat. Mekh.* **20**, 5, 669–671, Sept.-Oct. 1956.

Author generalizes the problem of buckling of a thin tube subjected to torsion by introducing both axial loading N_x and uniform lateral pressure q. He extends the equations for the displacements given in Timoshenko's "Elastic stability." He uses the same method to derive the equation for the critical stress.

In the generalized case, the resulting equation must be analyzed numerically. This is done for the particular case of an infinitely long circular cylinder with closed ends subjected to torsion and internal pressure q. The critical torsional stress t is computed and its value is tabulated for various radius/thickness ratios and several values of q. The results show that the critical t increases very substantially as q increases. Also, for the limit radius/thickness $= \infty$, q and t are linearly related.

G. H. Beguin, USA

2533. Herber, K.-H., Simplified, unified calculation of buckling problems for bars with fixed ends and for frames. Parts I, II, III (in German), *Bautechnik* **33**, 5, 157–160, May 1956; **33**, 9, 326–329, Sept. 1956; **33**, 12, 435–442, Dec. 1956.

Author discusses buckling problems in civil engineering structures with special reference to German building code (DIN 4114) requirements and methods of analysis. Approximate methods for determining real buckling load of bars with axial and transverse loading, one-story single- and multiple-span building frames and symmetrical two-story frames are presented. Effect of twist-buckling on buckling loads is considered. Worked examples are included.

C. Birnstiel, USA

2534. Bijlaard, P. P., Buckling of plates under non-homogeneous stress, *Proc. Amer. Soc. civ. Engrs.* **83**, EM 3 (*J. Engng. Mech. Div.*), Pap. 1293, 32 pp., July 1957.

Plastic buckling stresses are calculated for long plates, clamped at their unloaded edges and subject to longitudinal bending or eccentric compression in their plane. As in the author's earlier work on simply supported plates to this end, the governing partial differential equation is reduced to ordinary finite-difference equations, using second-order differences. The plastic buckling stresses of clamped and hinged flanges are calculated by the energy method. For clamped and hinged flanges also the elastic buckling stresses are calculated. Simple design formulas and graphs are presented for the plastic reduction factors with which the buckling stresses for the associated elastic cases have to be multiplied for obtaining the plastic buckling stresses.

From author's summary by B. Budiansky, USA

2535. Guest, J., and Solvey, J., Elastic stability of rectangular sandwich plates under bi-axial compression, *Aero. Res. Lab. Rep. Melbourne* SM-251, 22 pp. + 14 tables + 23 figs., May 1957.

Theoretical values of critical loads are presented numerically for plates having (1) all edges clamped, and (2) two opposite edges clamped and two simply supported, over varying ratios of (1) biaxial loads, (2) plate dimensions, and (3) stiffnesses of core and facings. The analysis minimizes strain energy with respect to forms of cross-sectional displacements less simple than rotations about middle-surface lines, then uses a single-term expression for a mode of $m \times n$ half-waves to obtain the critical loads by virtual work. Interactive effects of biaxial loads are shown explicitly in the many tables and graphs.

L. Maunder, Scotland

2536. Popov., S. M., Stability beyond the limits of elasticity of rectangular plates in eccentric tension or compression (in Russian), *Inzbener. Sbornik, Akad. Nauk SSSR* **18,** 165–173, 1954.

An eccentric distributed tensile or compressive load in the xy plane on a plate whose middle surface is also in the xy plane can be replaced by (is equivalent to) a tensile or compressive resultant concentrated force and a resultant moment about z axis. Author solves the title problem and finds the critical values of the resultant concentrated force and the resultant moment. The solution involves long and complicated derivations.

T. Leser, USA

2537. Gerard, G., Plastic stability theory of thin shells, *J. aero. Sci.* **24,** 4, 269–274, Apr. 1957.

Donnell-type equations are formulated for buckling of shells of constant unequal principal radii, with stiffnesses in the plastic range calculated on the basis of simple deformation theory with no unloading in the shell. Plasticity reduction factors, to be applied to the elastic buckling load, are calculated for spherical shells under external pressure, circular cylinders subjected to lateral pressure, and circular cylinders in torsion. Small-deflection theory is used throughout, but, in the case of the spherical shells, it is proposed that the plasticity reduction factors thus calculated be applied to the more accurate results for elastic buckling predicted by large-deflection theory.

B. Budiansky, USA

2538. Hodge, P. G., Jr., and Nardo, S. V., Carrying capacity of an elastic-plastic cylindrical shell with linear strain-hardening, *Mech.* **25,** 1, 79–85, Mar. 1958.

An approximate solution is given for the hydrostatic external pressure required to collapse a simply supported cylindrical shell loaded both radially and at its ends. A bi-linear stress-strain diagram is assumed, with a special yield condition. The principle of minimum potential energy, shown by Hodge in earlier papers to be applicable, is used to find stresses and displacements. Results are compared in an example to collapse pressures predicted by rigid-plastic and elastic theories. The elastic-plastic collapse load is considerably less than either.

Reviewer believes it would have been worth while to explain how the yield condition used is related to either the well-known theories of strength or to the behavior of real materials, and to have compared the solution with simpler elastic-plastic approaches such as Sturm's ["A study of the collapsing pressure of thin-walled cylinders," *Univ. Illinois Exp. Sta. Bull.* no. 329, Nov. 1941].

A. D. Topping, USA

2539. Archer, R. R., Stability limits for a clamped spherical shell segment under uniform pressure, *Quart. appl. Math.* **15,** 4, 355–366, Jan. 1958.

Author gives an approximate solution of the subject nonlinear boundary-value problem using perturbation methods. His solution is based upon a one-parameter *deflection/thickness* expansion and he is able to obtain solutions for a greater range of shell geometries than Kaplan and Fung [AMR **8** (1955), Rev. 2330]. Agreement between theory and experiment is still relatively poor in most cases, however. Pressure considered by author is said to be radial. Since experimental results are for hydrostatic loading, reviewer wonders whether part of discrepancy could be explained by phenomenon found for ring buckling [centrally directed loading having a critical load 50% higher than the hydrostatic case]. Other possible sources for the discrepancy are initial imperfections, inelastic buckling and disturbances during testing. Author suggests the last possibility.

Those interested in this problem can find additional material in the *Journal of aeronautical Sciences* [July '57, Feb. '58]. These analyses, based on two-parameter expansions, give results which are somewhat different from those of the present author.

G. D. Galletly, USA

2540. Bernstein, M. S., A solution of the problem of the stability of a circular ring by the energy method (in Russian), *Naucb.-tekbn. inform. Mosk. lesotekbn. in-ta* no. 7, 8–16, 1955; *Ref. Zb. Mekb.* no. 11, 1956, Rev. 7835.

The critical values of the distributed radial and normal loads on a circular ring are determined; in both cases they lead to the well-known formula of Maurice Levy. The solution is obtained by the energy method. An original device is used in calculating the work done by the external load: the ring is regarded as the limiting case of a hinged polygon, loaded at the joints by a system of balanced forces.

N. A. Alfutov

Courtesy Referativnyi Zburnal, USSR

Translation, courtesy Ministry of Supply, England

2541. Ceradini, G., Elastic instability of arches under stresses and deformations in their plane (in Italian), *G. Gen. civ.* **94,** 10, 687–708, 1956; **94,** 11/12, 781–821, 1956.

Modern use of structural materials of high strength, such as prestressed concrete, steel and alloys with great resistance, especially in bridges, skyscrapers, ships, long-span space structures, aircraft, etc., has made thorough knowledge of elastic instability very important. Several decades ago S. Timoshenko investigated elastic instability of circular arches and curved bars with small curvature. Thorough analysis is presented of arches with one, two, and three hinges, restrained, circular, parabolic, with constant and variable moment of inertia. First part deals with application of static criterion, influence of axial deformation; part II with energetic criterion applied to parabolic arches of second and fourth degree with constant and variable moment of inertia, subject to concentrated and uniform loading, symmetrical and asymmetrical deformation; part III discusses the title subject under consideration of the elastic theory of the second order. This part includes definitions of fundamental hypotheses and describes influence of deformations on entity of forces to which arch elements are exposed, virtual work of these forces and their potential energy, re-

lations of orthogonality, solution methods of equations of the indifferent equilibrium, solution of the elastic problem of second order, etc. J. J. Polivka, USA

2542. Bolotin, V. V., On the errors in some works on dynamic stability (in Russian), *Izv. Akad. Nauk SSSR, Otd. tekh. Nauk* no. 11, 144–147, 1955; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8543.

Some errors and inaccuracies are noted in articles by F. Weidenhammer [*Ing.-Arch.* **18**, no. 6, 1950], V. E. Salion [*Dokladi Akad. Nauk, USSR* no. 5, 1950, and others], and V. A. Sobolev, devoted to dynamic stability of elastic bars.

G. Yu. Dzhanelidze
Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

Joints and Joining Methods

(See also Revs. 2456, 2515, 2597, 2628, 2798)

2543. Sanks, R. L., and Rampton, C. C., Jr., Grit and shot-reinforced high tensile bolted joints, *Proc. Amer. Soc. civ. Engrs.* **83**, ST 6 *(J. Struct. Div.)*, Pap. 1435, 32 pp., Nov. 1957.

Paint in high tensile bolted joints drastically reduces frictional resistance to slip. Grit and shot placed between painted contact surfaces increases frictional resistance. Shot-reinforced painted joints are, in static tests, equal to or somewhat better than clean mill-scale joints, and much superior to riveted joints.

From authors' summary

2544. Heywood, R. B., Simplified bolted joints for high fatigue strength, *Engineering* **183**, 4744, 174–178, Feb. 1957.

2545. Lake, G. F., and Boyd, G., Design of bolted flanged joints of pressure vessels, Instn. mech. Engrs., Prepr. 17–32, 1957.

2546. Bluhm, J. I., and Flanagan, J. H., A procedure for the elastic stress analysis of threaded connections including the use of an electrical analogue, *Proc. Soc. exp. Stress Anal.* **15**, 1, 85–100, 1957.

Screw thread failure stresses depend on load distribution and tensile fillet design. Equations are derived and analogue proposed for determining load distribution. Tensile fillet stresses are determined by photoelasticity and by equation based on photoelasticity. Design considerations of efficient threads are presented.

C. R. Freberg, USA

2547. Francis, A. J., and Belcher, G. L., Tests on eccentrically loaded riveted joints, *Aero. Res. consult. Comm., Aero. Res. Lab., Melbourne, Austral.*, SM 249, 14 pp. + 1 table + 14 figs., Apr. 1957.

2548. Mordfin, L., and Legate, A. C., Creep behavior of structural joints of aircraft materials under constant loads and temperatures, *NACA TN* 3842, 53 pp., Jan. 1957.

The results of 55 creep and creep-rupture tests on structural joints are presented. Methods are described by which the time to rupture, the mode of rupture, and the deformation of joints in creep may be predicted. These methods utilize creep data on the materials of the joint in tension, bearing, and shear. The accuracy of these methods is within the scatter of the materials creep data.

From authors' summary

2549. Jessome, A. P., The efficiency of scarf joints, Canad. Woodworker, 4 pp., June 1956.

The laminations of glued structural members can be end-jointed by using glued hooked and end-stepped scarf joints. The notch effect of such joints can reduce the efficiency of the individual lamination more than 40% in tensile-parallel-to-grain strength due to stress concentrations, according to tests performed at the Ottawa Forest Products Laboratory. E. Stern, USA

2550. Eickner, H. W., General survey of data on the reliability of metal-bonding adhesive processes, *U. S. Dept. Agric., For. Prod. Lab. Rep.* 1862, 6 pp. + 1 table + 1 fig., May 1957.

A survey was made among a number of representative aircraft fabricators and adhesive manufacturers to obtain information on the reliability of adhesive-bonding processes for metals based on the variability of lap-joint strength data from tests for adhesive acceptance, laboratory control, adhesive storage, and production control. Practically all tests were on standard lap-joint specimens at room temperatures. The range of the coefficient of variation for these metal-bonding tests was generally from 6.4 to 16.2% with most of the adhesive-storage and production-control data in the range above 10%. From the data obtained, there were no indications that the coefficient of variation for the strength data was greater for one type of adhesive than for another.

From author's summary

2551. Petrov, I. P., Influence of a complex stressed state on the deformation and strength characteristics of the metal of pipes and welded joints (in Russian) *Trudi Vses. n.-i. in-ta po str-vu ob'ektov neft. i gaz. prom-sti.* no. 6, 5–30, 1954; *Ref. Zh. Mekh.* 1956, Rev. 5549.

A description of a method and results of tests of thin-walled pipes of low carbon steel (0.14–0.17% C) subjected to elongation at a given value of the internal pressure. Pipes of 76-mm diam, without welded seams, and pipes of 89-mm diam with an annular welded seam were tested. The pipes were brought to destruction by an axial load at three values of the internal pressure of 50, 100 and 150 atm, which created annular stresses of approximately 10, 20, and 30 kg/mm^2, respectively, in the wall of the pipe. Deformation in the axial and tangential directions and at an angle of 45° to the axis of the pipe was recorded with the aid of wire-resistance extensometers and lever-indicating tensometers. The positions of the yield point (appearance of the flow stage) and the yield strength were fixed from the diagram and the readings of the testing machine.

The values obtained for the yield and strength limits are compared with those calculated by the energy theory of strength. Data are given from which it follows that the increase in the longitudinal stresses at the yield point caused by the application of internal pressure amounts to 3–5% as opposed to 10–15% according to the energy theory of strength. Values of the strength limit of the basic metal and of welded pipes are given in relation to the value of the annular stresses. The nature of the destruction of the pipes is described. S. A. Ratner, USSR

Courtesy Referativnyi Zhurnal
Translation, courtesy Ministry of Supply, England

2552. Alekin, L. E., Structure and method of analysis of the process of self-regulation of an arc in welding (in Russian), *Trudi sektsii po nauch. razrabotke probl. elektrosvarki i elektrotermii Akad. Nauk, SSSR*, no. 1, 69–89, 1953; *Ref. Zh. Mekh.* no. 10, 1956, Rev. 6443.

Construction of a structural arrangement corresponding to the process of self-regulation occurring during electric arc-welding. A table is made of the components which make up this arrangement and their characteristics, coefficients and time constants, and the points of application of the disturbing effects are explained. Expressions for the transmission functions and time characteristics are formulated.

On the basis of the analysis of these expressions certain recommendations are given in respect of individual parameters of the

system and an example is given of the calculation of the static errors and of the transitional processes.

I. M. Smirnova

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2553. Meredith, R., and Baird, B. L., Design and techniques for arc welding titanium, *Prod. Engng.* (Design Digest Issue) **28,** 15, G28–G31, Oct. 1957.

2554. Levine, M., Quick-release fasteners, *Prod. Engng.* (Design Digest Issue) **28,** 15, G34–G35, Oct. 1957.

2555. Subcommittee III, ACI Committee 325, Structural design considerations for pavement joints, *J. Amer. Concr. Inst.* **28,** 1, 1–28, July 1956.

Considerations are presented for the structural design of joints in concrete pavements for highways and airports. A description, function, and classification of joints; assumptions and materials to be used; and joint design details are included. Special consideration is given to applicable design criteria for tie bars and dowels. From authors' summary

2556. Cost of fastening assembly operations, *Prod. Engng.* (Design Digest Issue) **28,** 15, p. G4, Oct. 1957.

Structures

(See also Revs. 2401, 2402, 2486, 2487, 2497, 2502, 2503, 2508, 2525, 2531, 2535, 2539, 2541, 2588, 2598, 2604, 2609, 2621, 2628, 2633, 2657, 2859, 2862, 2863, 2870, 2878, 2906)

Book—2557. Abbett, R. W., edited by, American civil engineering practice, Vol. III, New York, John Wiley & Sons, 1957, xiii + 1263 pp. + index. $15.

The third volume of this valuable reference book—the first two volumes of which (see AMR **10** (1957), Rev., 1074) have become well known and appreciated in the first year after publication—has thirteen sections devoted to: theory of structures; masonry and plain concrete; reinforced concrete; prestressed-concrete structures; footings, piers and abutments; retaining walls; steel bridges; reinforced-concrete bridges; buildings; steel towers, masts, tanks, bunkers, bins; light-gage steel construction; reinforced-concrete chimneys, silos, bins, elevators, tanks, and turbine foundations; timber structures; earthquakes and earthquake-resistant design.

As stated in the preface by the editor, book seeks to "present the fundamental principles, procedures and data of modern civil engineering in concise form for reference purposes, with illustrations from current practice." This aim has been attained in a very useful manner. The point at which a given theory or procedure has to be considered as "practice" has been selected with extreme care; only methods and findings tested by experience are listed, but in these the material presented is up to date. Book does not go into details on special topics (e.g., theory of shells). Efforts are made to present European as well as American methods (e.g., presented concrete structures). Reviewer believes that the book is a very valuable contribution in developing internationally accepted unified design methods; this, in spite of the difference between English and metric measuring units.

Book is exceptionally well edited: the work of the contributors—many distinguished engineers—has been moulded with great care and precision into one unit. The illustrations are numerous, well-selected, and precisely constructed; in many fields, numerical examples worked out in detail facilitate the use of the book. Reviewer believes that the listing of some important references and standard books, and recommendations for further study at the end of each section would add to the usefulness of the book.

A. Kezdi, Hungary

Book—2558. Wang, C. K., and Eckel, C. L., Elementary theory of structures, New York, McGraw-Hill Book Co., Inc., (Civil Engng. Series), 1957, ix + 387 pp. $7.50.

Beginning with general description about the equilibrium of coplaner force system, simple but fundamental theory of a beam is given in which concepts of shearing force and bending moment are explained. Problems concerning only tension and compression, such as general truss, roof truss and building bents, are treated in the next four chapters. After explaining the influence diagram for simple beam and truss, and the general theories about the action of moving load on beam, important points about railway bridge with regard to its design analysis are described. In the next chapter, loads necessary to design highway and railway bridges are explained; this chapter is an introduction to their design method.

Statically indeterminate structures are discussed in the last five chapters. To insure better understanding, authors make clear the properties of statically indeterminate structure by means of its deflection, which leads to a method of its analysis.

Of all methods of analysis, emphasis is put on the slope deflection and the moment distribution methods, which are the most prevailing ones in use now. Numerical examples are ample enough to give sufficient understanding of each item, and brief but clear descriptions show the relationship between theories and their applications to practical design problems. T. Mogami, Japan

Book—2559. Heyman, J., Plastic design of portal frames, New York, Cambridge University Press (Cambridge Engineering Series), 1957, viii + 104 pp., $2.

This is an elementary, but rather exhaustive, study of the plastic carrying capacity of single-story, simple or multiple-bay portal frames under proportional loading. Since it is apparently intended for direct practical use by undergraduate engineers concerned with the "how" rather than the "why," it describes the method of analysis for the plastic collapse load of such frames in great detail, with numerous examples, without considering the well-known limitations of this approach with respect to deflection and stability. The extensive bibliography completely disregards the fact that the described method had been *fully* developed in the early twenties by Fritsche, Girkmann, Meier-Leibnitz and many others, as can be easily seen by perusing the Reports of the International Congresses for Bridge and Structural Engineering *before* World War II; not a single one of these names appears in the bibliography.

A. M. Freudenthal, USA

2560. Zimmermann, R. Z., Jr., The design of rigid frame bents, *Proc. Amer. soc. civ. Engrs.,* **83,** ST 6 (J. Struct. Div.), Pap. 1434, 39pp., Nov. 1957.

Paper purports to compare moment-distribution, column-analogy and slope-deflection solutions of simple rectangular portal frame subjected to unsymmetrical loading and thus involving sidesway. Topic is trivial and better treatments have long been available in most elementary books on indeterminate structures. Apparently author is unaware that every technique used in simplifying a moment-distribution solution is also applicable to a slope-deflection solution. His slope-deflection analysis is accordingly far more complicated than need be and consequently is prejudicial.

J. E. Goldberg, USA

2561. Michalos, J., and Louw, J. M., Properties for numerical analyses of gusseted frameworks, *Bull. Amer. Rly. Engng. Assn.* **58,** 530, 1–51. June–July, 1956.

In typical trusses considerable secondary stresses are produced by haunched and gusseted joint connections. The stability of rigid frameworks depends on restrained joints with haunches or

gussets. In both cases the rigidity of individual structural members is increased and consequently the bending moments are greater than in members having constant moment of inertia throughout their whole length (as is usually assumed in typical stress analysis). Numerical method of moment distribution is used which makes the analysis more complicated, since the accuracy can be checked only after the complicated calculation is completed. Other more practical methods can be used for the effect of haunches and gussets. In this report the effect of axial loads on flexural properties is included, but simpler charts are also presented for the more usual cases in which this effect is negligible. Two types of analysis are illustrated by examples: railroad bridge truss and a floor beam hanger frame. The resulting increase in stresses is particularly significant in considering fatigue.

J. J. Polivka, USA

2562. Anevi, G., Determination of crippling stresses for curved frames of alclad 24S-T in bending (in English), *SAAB Aircr. Co. Linköping, TN* 36, 73 pp., 1957.

2563. Hajnal-Konyi, K., The use of high-tensile steel reinforcement, *Concr. constr. Engng.* **52,** 12, 397–408, Dec. 1957.

2564. Kogan, L. A., The construction of influence lines for the stresses in three-dimensional framework (in Russian), *Trudi Uralsk. politekhn. in-ta* no. 54, 23–43, 1955; *Ref. Zh. Mekh.* no 11, 1956, Rev. 7845.

The method under examination for constructing the influence lines of the stresses in three-dimensional frame systems is a development of the method applicable to flat trusses by the technique of distributed constraining moments. Influence-line constructions are shown for the end moments, bending moments in the span, longitudinal and transverse forces. E. I. Silkin

Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

2565. Pikovsky, A. A., and Popov, N. I., The method of displacements (in Russian), *Trudi Rostovsk. in-ta inzh. zh.-d. transp. no.* 19, 150–172, 1955; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7838.

Book—2566. Kinney, J. S., Indeterminate structural analysis, Reading, Mass., Addison-Wesley Publishing Co. (Civil Engng. Series), 1957, xiii + 655 pp. $9.50.

This new addition to the abundant literature on statically indeterminate structures follows in general the American tradition and covers in particular the matters taught in American undergraduate courses. After two chapters devoted to basic concepts such as determinateness, principle of superposition, principle of virtual displacements, Maxwell's and Betti's reciprocal theorems (chapters 2 and 3), there follows a very long exposition of the methods for computing deflections (chapter 4) which covers more than 110 pages. Then comes the presentation of the methods for solving statically indeterminate structures: 1. The general method (or method of consistent displacements), due to Maxwell, Mohr and Muller-Breslau (chapter 5). 2. The method of least work (chapter 6). 3. The column analogy (chapter 7). 4. The method of moment distribution (chapters 8 to 10). 5. The slope deflection method (chapter 11). 6. Influence lines (chapter 12). 7. Elastic arches (chapter 13).

In addition to these more or less conventional matters, the book contains two unusual features: a brief history of structural theory starting from the prehistoric ages (chapter 1) and completed by historical notes scattered throughout the text, and a chapter on the model analysis of structures (chapter 14).

It is difficult for a European teacher to evaluate such a work without being unduly influenced by the general trend of teaching in Europe, which usually aims at stressing the fundamental principles in a very general way. The meaning of this remark will be understood more clearly if the reader compares the book under review with a book recently published in Switzerland [F. Stüssi's "Baustatik," volume 2], which can be said to represent the classical point of view in Europe.

Considered from this European viewpoint, it seems that the methods for computing deflections are overextended; all possible techniques are exposed, including an interesting extension (due to the author) of Mohr's conjugate beam method to a new conjugate structure technique. It seems to this reviewer that Müller-Breslau's general expression for a deflection $\delta_{ab} = \int Mm/EI\, ds$ (eventually extended to cover extensional, shear or torsional deformations) covers the whole subject and can be applied systematically. It seems also that the juxtaposition of five methods for solving statically indeterminate structures must leave the reader somewhat confused; more emphasis should be placed on the connections between these methods; as an example, among many, the general method and the method of least work are only two different presentations of the same thing: the unknowns are the same in both cases and the numerical calculations can be shown to be identical. Beyond the detailed treatment of particular examples, a systematization of the general method, containing the general equations, a discussion of the best choice of unknowns for reducing to zero a maximum of coefficients δ, practical methods for solving the system of linear equations, eventual use of a statically indeterminate fundamental system, etc., should be useful. On the other hand, a European engineer would be interested to know that the Hardy Cross method of moment distribution is convergent and that the error after a definite number of balancing operations is less than a definite quantity; he would like to know in which order the joints must be released in order to obtain the quickest convergence.

The treatment of arches, as usual, is somewhat superficial. As a matter of fact, it is hardly possible to cover all aspects of the theory without entering into details of erection procedures.

Extended references are furnished at the end of each chapter, but they are a little one-sided. For instance, in the chapter on models, a detailed presentation is given of Beggs' and Eney's techniques, but nothing is said about the Magnel influentiometer, which is considered in Europe to be a definite improvement over Beggs' apparatus. Besides these criticisms, which refer mostly to matters of personal taste, it should be emphasized that the book has outstanding qualities: it is clearly written and nicely presented, profusely illustrated with numerical examples, many of which are up-to-date and will certainly constitute a new reservoir of problems for teachers. At the end of the book, six pages of charts give the fixed-end moment, stiffness and carry-over factors for members with straight or parabolic haunches at one end or at the two ends. C. Massonnet, Belgium

2567. Zotin, I. P., The calculation of multistorey trusses (in Russian), *Trudi Uralsk. politekhn. in-ta* no. 54, 69–81, 1955; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7843.

A method is developed of distributing the constraining moments in the calculation of multistory, multispan, flat trusses.

A brief description is given of a method for calculating multistory multiply-tired frameworks of any degree of complexity, under horizontal and vertical loading. An approximation formula is presented for determining the horizontal displacement of any tier in a multiply-tiered, multistory truss, by the action of horizontal joint forces. A. F. Anishchenko

Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

Book—2568. Proceedings of the world conference on earthquake engineering, Berkeley, Calif., Dept. Engng., Univ. Extension, Univ. of Calif., 1956, $8.50. (Paperbound)

Forty two papers (exclusive of introduction, etc.) presented at the International Conference are included without discussion of them. Because of the variety of subjects, the book is a good reference in which to see the trends in eleven nations during the past fifty years. The contents include:

1. Ground motion (4 papers): Seismicity, seismometer and response spectrum technique.
2. Analysis of structural response (11 papers); various methods such as limit-design, linear or nonlinear vibration, and probability theory, etc., are applied.
3. Experience and practice (11 papers): Damages and aseismic codes, etc., in various nations.
4. Effects on soil and foundation (5 papers): Dynamic properties of soil and ground, and design of foundation.
5. Examples of aseismic design (6 papers): Quay wall, dam, building and tower, etc.

Finally, two panel discussions are reported briefly.

I. Toriumi, Japan

2569. Tokar, R. A., The problem of the safety factor in the stability analysis of building foundations (in Russian), *Trudi n.-i. in-ta osnovaniy i fundamentov* no 24, 31–38, 1954; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7803.

Author suggests determining the coefficients of stability of a structure by the ratio of the critical load to that actually existing. This is not a new suggestion [cf. B. N. Zhemochkin, "The calculation of embankment slopes." Steklografiya MIIT, 1928; M. M. Grishin, Gidrotekhn. str-vo, 12, 1936]. K. A. Ksenofontov

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2570. Aichinger, H., On the amplification of the theory of open prismatic structures, with rigid edge joints, consisting of small prismatic slabs (in German), *Bauingenieur* **30**, 11, 397–403, Nov. 1955.

Amplification is made in two different ways: (1) Deduction of the two systems of equations for the unknown shear stresses and transversal bending moments in the edges, supposing an arbitrary longitudinal distribution of loads, is realized by "influence factors." Continuous structures over various transversal slabs (tympans) and with variable thickness in the cross section were considered too. (2) Conditions of equilibrium and deformation were deduced for prestressed boundary slabs.

Author suggests the presented amplifications are specially useful for approximate computation of cylindrical shells of variable thickness and noncircular section curve, substituting the curved shell by a polygonal prismatic structure. Though numerical work for practical calculation is considerable, application of described method is sometimes the unique possibility for computing the stress distribution in this type of structure. No practical example is given.

H. Beer, Austria

2571. Sonntag, G., Corrugated-web girder (in German), *Holz-Zentralblatt, Stuttgart*, no. 86, July 1956

An I-shaped girder with grooved lumber top and bottom chords and a corrugated plywood web—inserted into and glued to the chord members—was investigated prior to its current mass production in Germany. The 1:12 ratio of length of corrugation parallel with the girder to width of corrugation is recommended. The size of the chord members and the thickness of plywood web vary with the size of the girder. A 50% saving in materials is the result of the use of this construction, if a comparison is made with an equivalent solid timber beam. Continuous production of any length is accomplished by means of fully automatic machines.

E. G. Stern, USA

2572. Willis, J., Some notes on sandwich design for minimum weight as applied to airplane wings, *Aero. Engng. Rev.* **16**, 10, 44–47, Oct. 1957.

2573. Cook, F. E., and Milwitzky, B., Effect of interaction on landing-gear behavior and dynamic loads in a flexible structure, *NACA Rep.* 1278, 20 pp. + append., 1956.

[See AMR 9, Rev. 44]

2574. Maksimadzhi, A. I., and Rudnev, N. N., Approximate formula for the determination of the torsion moment of the hull of a ship (in Russian), *Trudi Tsentr. i-n. in-ta, mor. flota* **1**, 1, 57–63, 1955; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8634.

A formula is evolved for the determination of the magnitude of the moment of torsion acting on the ship on its course angularly to the wave. The following assumptions are made: (1) the wave's profile is sinusoidal, (2) the height of the wave is expressed by the relation $b = 1/30\lambda + 2$, where λ is the length of the wave in meters, (3) the calculated length of the wave $\lambda = L \cos \phi$, where L is the length of the ship, ϕ is its course angle relative to the wave, (4) the cross section of the ship is parabolic, (5) the sides are straight-walled, (6) the vessel's ascent of the wave is stable.

It was established as the result of the investigation that (1) the course angle corresponding to the maximum moment of torsion equals 62–77° and (2) that the ratio of length to width of the ship has little effect on the magnitude of the moment.

V. S. Chuvikovskii

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

Rheology (Plastic, Viscoplastic Flow)

(See also Revs. 2483, 2488, 2537, 2538, 2548, 2559, 2587, 2614, 2621, 2627, 2630, 2647, 2783, 2861)

Book—2575. Prager, W., Problems of theoretical plasticity, Paris, Dunod, 1958, 121 pp. 1650 fr.

This is essentially an unchanged version of the previous German edition [AMR 9 (1956), Rev. 1094], except that the literature references have been brought up to date and a few errors or ambiguous statements have been corrected. From the preface

2576. Basinski, Z. S., The instability of plastic flow of metals at very low temperatures, *Proc. roy. Soc. Lond. (A)* **240**, 1221, 229–242, May 1957.

The low-temperature unstable plastic deformation of aluminum alloys is described. It is shown that discontinuities in the stress-strain curve are caused by a localized temperature rise produced during the deformation. The calculated magnitudes of the drops in load and the transition temperature between smooth and discontinuous flow agree reasonably well with the experimental observations. It is believed that all metals should exhibit unstable deformation at sufficiently low temperatures.

From author's summary

2577. Olszak, W., Perzyna, P., and Szymanski, C., Two-dimensional problems in the theory of plasticity of nonhomogeneous anisotropic bodies (in English), *Arch. Mech. stos.* **9**, 2, 335–358, 1957.

The principal part of the paper concerns the analysis of conditions connected with the appearance of two-dimensional states assuming anisotropy and nonhomogeneity of the general type.

Four different definitions are given for two-dimensional state of strain of anisotropic nonhomogeneous bodies. The first, which is the most general, assumes that the strain increments depend only on two coordinates x_1, x_2. Then "stronger" assumptions are

made: (2) $d\varepsilon_{33} = 0$; (3) a plane state of strain, that is we have also $d\varepsilon_{13} = d\varepsilon_{23} = d\varepsilon_{31} = d\varepsilon_{32} = 0$; (4) the same assumptions but with an additional limitation of the stress tensor to normal stresses and tangential stresses parallel to the basic plane (classical definition). From these definitions general conditions for obtaining one of the two-dimensional states mentioned are obtained.

The consequences of the general equations introduced in the first part, if the material has a monoclinic, orthotropic, transversally isotropic, cubic or isotropic structure, are investigated. Yield conditions and equations for the components of the strain tensor are established. The cases where states corresponding to the classical definition appear are indicated.

States of stress are also considered.

W. Urbanowski, Poland

2578. Lepik, Yu. R., Strength of rectangular, elastic, and plastic plates nonuniformly compressed in one direction (in Russian), *Inzhener. Sbornik, Akad. Nauk SSSR* **18**, 161–164, 1954.

All stability problems of elasto-plastic plates solved up to now assume uniform stress before the loss of stability. Author of this paper solves a problem of nonuniformly compressed plate with additional assumption that the material of the plate is noncompressible. He applies the theory given by A. A. Ilyushin, "Plasticity," OGIZ, 1948 (in Russian) [AMR **6** (1953), Rev. 464] and his derivations can be followed only with Ilyushin's monograph available.

T. Leser, USA

2579. Ernst, G. C., Plastic hinging at the intersection of beams and columns, *J. Amer. Concr. Inst.* **28**, 12, 1119–1144, June 1957.

Principal object of investigation was to determine the amount of concentrated plastic rotation developed at the connection between beams and columns. Thirty-three tests were conducted, some at a slow loading rate, some at a fast loading rate equivalent to that prescribed for concrete cylinders. Steel ratios of 0.01, 0.03 and 0.05, column widths of 6, 12, 18, 24, and 36 in., and nominal concrete strengths of 3000 and 4000 psi were used.

Concentrated plastic rotation at concrete crushing and at maximum moment is markedly reduced when the steel ratio exceeds 0.01, and is also less for a fast loading rate. Theoretical moments agreed satisfactorily with experimentally determined moments in all cases. At concrete crushing for the 0.05 steel ratio under fast loading, concentrated plastic rotation was virtually negligible. It seems necessary, therefore, to consider the effect of steel ratio and rate of loading in cases for which the ultimate capacity of a structure is dependent upon a redistribution of moment produced by concentrated plastic rotations.

From author's summary

2580. Finnie, I., Creep buckling of tubes in torsion, *J. aero. Sci.* **25**, 1, 66–67 (Readers' Forum), Jan. 1958.

Author suggests that the deformation theory approach of Stowell and Gerard to plastic buckling is applicable to creep buckling when a time-dependent secant modulus is substituted. Test data "lend support to the hypothesis that torsional buckling should occur at shear strains depending only on the specimen dimensions and independent of stress or the material tested." Although such an assumption may serve as a rough guide, reviewer is not convinced the approach is a truly valid one, especially in view of the author's statement that "the critical strain predicted by the secant modulus may underestimate the strain at which creep buckling becomes significant."

D. C. Drucker, USA

2581. Kennedy, A. J., Problems of combined creep and fatigue design, *Engineer, Lond.* **204**, 5305, 444–447, Sept. 1957.

Article reviews evidence relating to interaction between processes of deformation and fatigue, particular attention being given to deformation by creep. Creep under various combined creep and fatigue stress conditions and effects of fatigue stresses on processes of recovery of work-hardening in discontinuous creep are discussed and illustrated by means of test results. It is shown that, whereas small and moderate fatigue stresses may strikingly increase recovery, fatigue stressing at high amplitudes may cause marked rehardening.

Nature of processes is discussed from the dislocation standpoint, and way in which more general methods may be developed is indicated.

F. J. Plantema, Holland

2582. Reiner, M., A centripetal-pump effect in air, *Proc. roy. Soc. Lond. (A)* **240**, 1221, 173–188, May 1957.

In considering the flow behavior of any fluid it is important to keep in mind the relative magnitudes of two rates: the rate of nonuniform flow and the rate of internal relaxation. Whenever the former rate becomes comparable with or is greater in magnitude than the latter, viscoelastic behavior may be expected. In particular, "normal stresses" will be experienced.

The work reported in this paper represents the first reported attempt at experimental verification of viscoelastic behavior of air. Although there is room for argument about the use of the Truesdell number to justify a fluid possessing an elastic shear modulus and various other details, there seems to be very little doubt that the observed effects are striking and significant and that they could have been due to the presence of "normal stresses."

Y.-H. Pao, USA

2583. Doolittle, A. K., and Doolittle, Dortha B., Studies in Newtonian flow. V. Further verification of the free-space viscosity equation, *J. appl. Phys.* **28**, 8, 901–905, Aug. 1957.

Values of the limiting specific volume v_0, calculated by a trial-and-error procedure from n-alkane viscosity data using the authors' free-space viscosity equation $\ln\eta = B(v_0/v_f) + \ln A$, agree substantially with previously published values obtained by extrapolation of density data. Values of v_0 for mercury, sodium, benzene, carbon tetrachloride, dichlorodifluoromethane, and water were calculated by the same trial-and-error procedure. In all cases the free-space equation gave a much better fit than the corresponding Andrade temperature equation, $\ln\eta = B/T + \ln A$.

From authors' summary

2584. Krasil'nikov, Yu. I., Unsettled motion of a viscous plastic liquid in a circular tube (in Russian), *Prikl. Mat. Mekh.* **20**, 5, 655–660, Sept.–Oct. 1956.

Failure, Mechanics of Solid State

(See also Revs. 2488, 2544, 2581, 2619, 2629, 2641)

2585. Weil, N. A., Bursting pressures and safety factors for thin-walled vessels, *J. Franklin Inst.* **265**, 2, 97–116, Feb. 1958.

The known condition of instability for tensile bars of plastic material is applied and supplemented for vessels under internal pressures with assumption of Hencky-Mises theory of plastic behavior. By adopting a parabolic expression for the true tensile stress-strain properties of the material it is shown that the safety factor for bursting of vessels under internal pressures depends only upon the strain-hardening exponent of the stress-strain line. The "ultimate tensile strength" from a normal uniaxial tensile test represents no real property of the material. The theoretical results are compared with experimental values.

L. Foppl, Germany

2586. Bueckner, H. F., The propagation of cracks and the energy of elastic deformation, ASME Ann. Meet., New York, N. Y., Dec. 1957. Pap. 57-A-189, 16 pp.

Paper deals with an extension of the well-known A. A. Griffith theory of 1929 in which the condition was defined under which a small crack in an elastic, but amorphous, substance (glass, silica chilled) will start to grow under the action of a two-dimensional (plane) state of stress. Griffith considered the decrease of the elastic strain energy due to the formation of a crack in a stressed body and the newly created capillary energy of the surface tension in the faces of the crack, the balance of these two types of energy supplying the critical condition for the growth of an existing, small tensile crack.

Author bases his ideas on certain assumptions attributed to E. Orovan and to G. R. Irwin who considered the condition for the propagation of a small crack in a polycrystalline material (mild steel) behaving elastically but capable of deforming also permanently, plastically. They assumed that in the critical condition for a polycrystalline substance the surface energy does not play a role, but that in a field of three-dimensional stress around the front edge of a growing tensile crack, a small more or less spherical plastic zone is continuously formed consuming deformation work that is lost in heat. They postulate that, in this manner, a thin layer of plasticized material is formed along the faces of a crack while it grows. However, this seems questionable because experiments (and the theory of the hyperbolic type of partial differential equations controlling the flow of a material that yields plastically) have shown that the plasticized zones may penetrate deeply into the stressed material along the layers of slip radiating out from the edge of the crack in two oblique directions. Author develops these ideas further for establishing criteria for the growth of small cracks in polycrystalline materials, considering the two cases when the externally impressed forces do not contribute to work if the grips are fixed or add to the work if the grips are movable in space, in both cases by assuming that the impressed forces are held at given, invariable values. He states that the energy supply for crack extension under constant load is larger with movable grips than under the condition of fixed grips. Like Griffith, he considers the decrease of elastic strain energy through the presence of a crack. Various such criteria are expressed in mathematical terms. A. Nadai, USA

2587. Cottrell, A. H., Deformation of solids at high rates of strain, *Chartered mech. Engr.* **4,** 9, 448–460, Nov. 1957.

Paper is the opening lecture delivered at the "Conference on Properties of Materials at High Rates of Strain," held at the Institution of Mechanical Engineers, London, in April 1957. The author, making extensive use of findings in the literature, presents a rather broad but well-integrated coverage of his subject. His theme is that molecular processes that exist in materials play a fundamental role in determining whether a strain rate is high or low. They act as pacemakers against which the applied rate of strain should be compared, and when the latter becomes large (measured in this way) important changes occur in the deformation process. This leads to a flexible classification of high and low rates of strain.

These molecular processes and their relation to strain rate are examined for specific dynamic deformation cases. Elastic and plastic waves, cracks in semibrittle materials, and heating effects form the first part of the paper. The remaining and larger part of the paper is devoted to a discussion of the dislocation theory of slip in a crystal, and the application of this theory in work on the yield point, yield and fracture at low temperatures, and delayed yielding. In connection with the latter, author introduces a new estimate of activation energy (associated with breakaway of a dislocation from its fixed ends) which shows good agreement with experiments of Clark and Wood [AMR 4 (1951), Rev. 232].

J. Miklowitz, USA

2588. Boyd, G. M., The conditions for unstable rupturing of a wide plate, *Trans. Instn. nav. Arch. Lond.* **99,** 3 (Part II), 349–366, July 1957.

Paper deals with a problem of great engineering significance in design of large steel structures, particularly in ships. The term "unstable rupturing" denotes the condition under which a fracture will extend under the sole influence of the elastic energy stored in the structure, i. e. without an increase of the external load.

Paper reviews the present position in the field as represented by the papers of Griffith, Greenspan, Mott, and others. Author proposes an extension to existing theories based on the assumption that fracturing of a wide plate after a transition period—under certain circumstances—approaches a "steady-state" condition.

All theoretical results and physical conclusions as stated in the paper are derived from the fundamental principle of conservation of energy as applied to the homogeneous, continuous and isotropic model-material of classical theory of elasticity. While the results of treating such a simplified model may well describe the physical aspects of fracture in noncrystalline materials, reviewer doubts that the characteristics of the brittle fracture of metallic media can be fully described or understood by such results. Crack initiation and propagation is taking place — step by step — in control volumes of the metallic materials for which a modulus of elasticity or quasi-isotropy cannot be defined. Reviewer's opinion is stressed by some contributions in the very extensive discussion at the end of the paper. A. Slibar, USA

2589. Drozdovsky, B. A., and Friedmann, Ya. B., The methodology of determining the crack sensitivity of materials (in Russian), *Zavod. Lab.* **21,** 5, 579–590, 1955; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7971.

The crack sensitivity is investigated of a high-strength steel ZOKhGSA, annealed and tempered at 200° or 510°, and heat-treated isothermally in various ways. Determination of the breakdown stress in static bending was made on notched samples, as well as on samples with cracks produced by preceding cyclic impact loading, and other methods.

In regard to crack sensitivity as determined by authors' method, low-annealed steel is found to be substantially worse, having a far lower value of specific work and breakdown stress than high-annealed steel. This peculiarity is not apparent in the values of the characteristics determined by other forms of mechanical testing, discussed in the paper. The presence is established of a limiting crack depth, in the particular case ~ 0.2 mm, at which the breakdown stress value of a crack-sensitive material (low-annealed steel) falls abruptly; further increase in the crack depth practically does not influence the value of the breakdown stress.

E. M. Shevandin
Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

2590. Avetissov, S. S., Erosion along streamlines in the ducts of a hydraulic turbine rotor (in Armenian), *Izv. Akad. Nauk Arm. SSR, Physico-Mathematical, Physical and Engineering Science Series* **8,** 5, 87–103, 1955; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7472.

An investigation of the wear by erosion of the rotors of an axial-type hydraulic turbine in one of the hydro-electric power stations of the Armenian SSR after a prolonged running period (32,345 hours) at varying speeds. It is demonstrated that for a qualitative assessment of the region of cavitational erosion, as well as for evaluation of the intensity and nature of the wear in different parts of the working ducts of the rotor of an axial hydraulic turbine, the profile theory can usefully be applied. S. N. Nasilov
Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

2591. Lioznyanskaya, S. G., Resistance of glass tubes to internal pressure depending on the magnitude of their residual stresses (in Russian), *Naucb.-tekbn. inform. byn. Vses. n.-i. in-ta stekla* no. 7, 29–36, 1954; *Ref. Zb. Mekb.* no. 12, 1956, Rev. 8742.

A study was made of the influence of a method of heat treatment on the stability of the tubes. The magnitude of the permissible residual tensile stresses was determined. The experiments were carried out on tubes under vertical tension, with inner diameters of 55 to 700mm and wall thicknesses of 4.5 to 5mm, after various heat treatments. It was shown that the residual stresses, when the condition prevailed of their even and symmetrical distribution, do not weaken the tube when deformations due to bending and elongation take place in the longitudinal plane of the tube.

V. V. Balandina

Courtesy Referativnyi Zburnal, USSR

Translation, courtesy Ministry of Supply, England

2592. Kobrin, M. M., Method of study of the stability of residual stresses when loaded cyclically (in Russian), *Zavod. Lab.* **21**, 4, 467–472, 1955; *Ref. Zb. Mekb.* no. 12, 1956, Rev. 8719.

A simple method is propounded enabling a decision to be made on the degree of removal of residual stresses in a smooth cylindrical test piece when friction is at work due to bending or torsion owing to change in length of the sample. The method is based on the analysis of the nature of deformation in the cylinder when residual stresses are removed—work carried out by L. A. Glikman [*Zavod. Laboratoriya* **9**,no. 2, 1940] ; the correctness of the data obtained by the method given above, is verified by a series of special check tests. The results are stated of the investigation of the influence of the magnitude of the frictional stress and the number of cycles on the stability of the residual stresses created by surface-hardening (peening).

V. A. Lomakin

Courtesy Referativnyi Zburnal, USSR

Translation, courtesy Ministry of Supply, England

2593. Jones, M. H., Newman, D. P., and Brown, W. F., Jr., Creep damage in a Cr-Mo-V steel as measured by retained stress ruptured properties, *Trans. ASME* **79**, 1, 117–126, Jan. 1957.

2594. Kepert, J. L., Patching, C. A., and Robertson, J. G., Fatigue characteristics of a riveted 24S-T aluminum alloy wing, Part I. Testing techniques, *Aero. Res. consult. Comm. aero. Res. Lab. Melbourne, Austral. Rep.* SM.246, 23 pp. + 8 tabs. + 17 figs., Oct. 1956.

Report describes two testing techniques which were utilized for the static and fatigue testing of Mustang P-51D wings.

A hydraulic loading rig was used for both the static and the high alternating load tests, while for the lower alternating load tests, in which the life exceeded 80,000 cycles, a resonant vibration rig was used in order to reduce the time of testing.

Details of the design and calibration of these rigs are given, together with results indicating that the methods of testing did not influence the fatigue life of the structure.

From authors' summary

2595. Weibull, W., Static strength and fatigue properties of unnotched circular 75S-T specimens subjected to repeated tensile loading, *Flygtekn. Försöksanst. Medd.* 68, 29 pp., 1956.

Data from a series of 24 static tensile tests and 270 fatigue tests have been statistically analyzed by determination of the median S-N curve, the scatter and the distribution function of the fatigue strength, as well as the P-S-N equation.

It was found that the scatter in fatigue strength is independent of the number of stress cycles applied and that the strength is neither normally nor log-normally distributed. The scatter due to the material is small and of the same magnitude as the scatter due to the testing machine.

Formulas for calculating the variances of estimated parameters have been derived and applied to the test data.

From author's summary

2596. Schijve, J., and Jacobs, F. A., The fatigue strength of aluminum alloy lugs, *Nat. LuchtLab. Amsterdam Rap.* M. 2024, 20 pp. + appendix, 3 pp. + 7 tables + 35 figs., Jan. 1957.

Fatigue diagrams have been established for unnotched specimens and two types of lugs of 2024-T aluminium alloy. The stress concentration factors for the lugs are 2.65 and 3.5 respectively. A survey of available data is given. Different factors which may effect the fatigue behaviour of lugs are discussed. Attention is drawn to fretting in lugs. Some remarks are made with respect to designing lugs for fatigue.

From authors' summary

2597. Starkey, W. L., Marco, S. M., and Collins, J. A., The effect of fretting on fatigue characterists of titanium-steel and steel-steel joints, ASME Ann. Meet., New York, N. Y., Dec. 1957. Pap. 57-A-113, 10 pp.

2598. Krishnan, A. A., and Maji, K. D., Fatigue failure—a structure study, *J. sci. indust. Res. India* **16**, 3, 105–119, Mar. 1957.

Fatigue failure in copper has been studied using x-ray back reflection and metallographic techniques. In the safe range, below the endurance limit, the structural changes are noticed up to a certain number of reversals, depending on the magnitude of stress. In the unsafe range there is a progressive change till at fracture it becomes maximum. Metallographic observations show that, in the unsafe range, the slip lines intensify with increasing stress cycles and that the cracks follow either slip bands or grain boundaries. The presence of crystallites at the root of a fatigue crack and the branching of cracks along crystallite boundaries have also been demonstrated. It is possible that the real answer to the mechanism of fatigue crack lies outside the scope of the x-ray and this microscope, viz. in dislocations.

From authors' summary

2599. Nordmark, G. E., and Eaton, I. D., Effect of fatigue crack on static strength: 2014-T6, 2024-T4, 6061-T6, 7075-T6 open-hole monobloc specimens, *NACA TM* 1428, 22 pp., May 1957.

Static tensile test results are presented for specimens of 2014-T6, 2024-T4, 6061-T6, and 7075-T6 aluminum alloy containing fatigue cracks. The results are found to be in good agreement with the results reported for similar tests from other sources. The results indicate that the presence of a fatigue crack reduced the static strength, in all cases, by an amount larger than the corresponding reduction in net area; the 6061-T6 alloy specimens were least susceptible to the crack and the 7075-T6 alloy specimens most susceptible. It is indicated that a 7075-T6 specimen may develop as little as one-third of the expected static tensile strength when the fatigue crack has consumed only one-fourth of the original area.

From authors' summary

2600. Gassner, E., The problem of fatigue strength in aircraft structures. A survey with recommendations for design rules based on recent research, *Aircr. Engng.* **28**, 329, 228–234, July 1956.

2601. Welter, G., and Dubuc, J., Fatigue resistance of simulated nozzles in model pressure vessels, *Welding J.* **36**, 6, 271s–274s, June 1957.

This paper brings together all observations concerning the fatigue resistance of simulated nozzles made during fatigue tests on model pressure vessels of A201, Grade A, and A302, Grade B steels under cyclic internal pressure. It was observed that all

fatigue fractures in these nozzles occurred in the longitudinal direction of the pressure vessels and began at points of maximum circumferential stress concentration at the edge where the nozzle hole meets with the internal surface of the pressure vessel shell. From authors' summary

2602. Rowe, R. E., An appreciation of the work carried out on fatigue in prestressed concrete structures, *Mag. Concr. Res.* **9,** 25, 3–8, Mar. 1957.

A review of the work carried out on fatigue in prestressed concrete is made, and observations by the various investigators are included.

All the tests show the excellent fatigue resistance of prestressed concrete in the design range of loading. From the review it is apparent that, for loadings outside the design range, i. e. after cracking has occurred, the governing factor affecting the fatigue strength of prestressed concrete is the fatigue strength of the high-tensile steel used for stressing. Figures obtained for the fatigue strength of high-tensile wires show that for most types in present use a mean value of 60–65% of the ultimate strength should be taken. No comparable figures for the fatigue strength of high-tensile alloy bars are available.

Suggestions are put forward for future research on fatigue in prestressed concrete and the need is emphasized for fundamental research into the behavior of both high-tensile wires and high-tensile alloy steel bars under fatigue conditions.

From author's summary

2603. Nordby, G. M., and Venuti, W. J., Fatigue and static tests of steel strand prestressed beams of expanded shale concrete and conventional concrete, *J. Amer. Concr. Inst.* **29,** 2, 141–160, Aug. 1957.

Tests on 27 beams cast from conventional and expanded shale aggregate concrete, prestressed with steel strand, are discussed. Fatigue tests at various load ranges and number of cycles of load repetition were performed on matched beams manufactured from both aggregates. Steel fatigue failures occurred in three specimens while the other 24 beams performed satisfactorily under fatigue loading. The three fatigue failures occurred when the beams were severely cracked during the repetitive loading; this failure was a result of stress concentrations and abrasion between the strands and the concrete. Those specimens undamaged by the fatigue loading were tested statically to failure, and either flexure or bond failures were recorded. The bond failures indicated that embedment length was the governing factor against failure rather than bond stress as computed from conventional equations. From authors' summary

2604. Duration of load and fatigue in wood structures: progress report of a sub-committee of the committee on timber structures of the structural division, *Proc. Amer. Soc. civ. Engrs.* **83,** ST 5 (*J. Struct. Div.*), Pap. 1361, 13 pp., Sept. 1957.

Paper summarizes present knowledge of the effects of long-time loading and of fatigue on structural wood. Long-time loading, whether continuous or intermittent, is considered from the viewpoints both of strength and of deformation. A chart showing the relation of safe working stress to duration of load is presented. Fatigue test data are reviewed, and representative *S-N* curves are shown. It is noted that fatigue failures are generally less of a problem in wood structures than in those constructed of other materials.

A section on the evaluation of old timbers in structures is included. It shows how old timbers can be appraised in terms of their species, grade, and condition to give a reliable estimate of their present structural value. From summary

2605. Lyons, W. J., Concerning the theory of fatigue failure in textile materials, *Text. Res. J.* **28,** 2, 127–130, Feb. 1958.

A minor modification of the reaction theory of flow is proposed with the aim of accounting, in terms of molecular processes, for the observed dependence of dynamic properties, particularly fatigue, on the frequency of stressing. To this end a factor $\phi(\nu)$, which is a function of the frequency ν, is introduced into the expression for the rate of breakage of bonds in the macromolecules of textile materials. It is shown that with this reasonable assumption of a frequency factor there results a general expression which, like earlier expressions, is compatible with the known dependence of fatigue life on temperature and stress under dynamic as well as dead loads. The present development goes further, permitting the derivation of a specific expression which is in accord with the observed frequency dependence of the fatigue life of a textile material.

From author's summary

2606. Bloomer, N. T., Time saving in statistical fatigue experiments, *Engineer, Lond.* **184,** 4783, p. 603, Nov. 1957.

2607. Marble, J. D., and Ruehrwein, C. V., Barrel finishing operation improves fatigue strength of jet engine parts, *Tool Engr.* **34,** 5, 99–101, Nov. 1957.

Material Test Techniques

2608. Davidson, R. W., Impact force-deflection diagrams for the FPL toughness test, *Forest Prod. J.* **8,** 1, 10–14, Jan. 1958.

An analysis is made of the internal forces—superimposed on the external forces—which exist during the performance of the toughness test as a result of the nature of the testing machine described.

A previously recommended improvement in the machine design, such as the substitution of a steel cable for the steel chain which transmits the impact load to the test specimen, should decrease the energy absorption by the intermediate member and, thus, result in more representative test data, in reviewer's opinion.

E. G. Stern, USA

2609. Andrejev, V., and Kostrencic, Z., Impact tests of columns in reinforced concrete (in Serbian), *Naše Gradevinarstvo* **11,** 12, 282–288, Dec. 1957.

Tests were performed on two models of reduced dimensions: cross section 10.8 × 10.8 cm (4¼ × 4¼ in.), height 68.9 cm (27 in.), resting on steel plate 12 × 12 cm (4¾ × 4¾ in.). Columns carry horizontal prismatic body in reinforced concrete 70.5 cm (28 in.) long, 19.6 cm (7¾ in.) wide, and 32.5 cm (13 in. deep), monolithically interconnected and topped with the same steel plate as at the bottom. Top body simulates the rigidity of supported floor structure. Hammer weighing 10 kg (23 lb) was dropped from five different heights, from 30 to 150 cm (from 12 to 60 in.). Two electric strain gages were installed and oscillations photographed in combination with Föppl's apparatus. Four hundred oscillograms demonstrated that the amplitudes of oscillation waves due to compression are smaller than those due to stresses. Tests results are compared with theoretical analysis as developed by S. Timoshenko. J. J. Polivka, USA

2610. Fine, M. E., Apparatus for precise determination of dynamic Young's modulus and internal friction at elevated temperatures, *Rev. sci. Instrum.* **28,** 8, 643–645, Aug. 1957.

Apparatus for determining Young's modulus and internal friction of metals at temperatures to 800 C is described. The resonant dynamic method with electrostatic transducers is used. The

frequency range is 20 to 200 kc/sec. Data for aluminum and tungsten are given. From author's summary

2611. **Wharff, P. C., Jr., and Crane, R. A., An automatic single-fiber tensile-strength tester,** ASME Ann. Meet., New York, N. Y., Dec. 1957. Pap. 57-A-158, 11 pp.

A machine has been developed to test single fibers, having several features of automatic operation when testing 30 to 40 samples an hour and plotting stress-strain diagrams which are corrected for cross section. A loading-chute enables the operator to only drop a sample in position without further manual adjustments. Electrical signals from the load circuit and elongation circuit are fed to an *X-Y* recorder. By means of a dernier correction the stress-strain curves are direct plots. Multiple range circuits enable one to obtain the initial plot and the total plots to rupture. Initial loading to 2% of the stress scale eliminates variations in the curves due to area and initial crimp. An additional correction is made for mounting variations. The denier for each sample is recorded on the graph. Sequence of operation is automatically controlled. More than 55,000 tests revealed the reliability of the machine and the ease in obtaining routine data that is very accurate and reproducible at room temperature and a set speed of testing. H. Majors, Jr., USA

2612. **Kochendorfer, A., and Wink, W., Influence of the property of the testing machine on the shape of the stress-strain curve in case of single-time and repeated loading** (in German), *Arch. Eisenhüttenw.* **28,** 2, 67-79, Feb. 1957.

Authors examine theoretically, for a testing machine, the influence on the force indicated by the measuring device and on the force really applied on the specimen, of the mass *m* of this device and of the factor *f* expressing the indicated force in function of the displacement of this measuring device, in the two cases of a single exceeding and of a repeated exceeding of the yield point. The theoretical results are compared with experimental results and satisfactory agreement is found.

Authors conclude that the properties of the testing machines influence appreciably the measured shape of the force-elongation diagrams. A faultless reproduction of the properties of the material is obtained with a machine for which the mass *m* is sufficiently small and the factor *f* is sufficiently large.

D. DeMeulemeester, Belgium

2613. **Reitzel, J., Simon, I., and Walker, J. A., New method for measuring linear compressibility of solids,** *Rev. sci. Instrum.* **28,** 10, 828-832, Oct. 1957.

An apparatus is described for measuring linear compressibility of rods and tubes under hydrostatic pressures to 4000 kg/cm² and temperatures to 260 C. The measurements are made by means of a linear-differential transformer located on the outside of the nonmagnetic high-pressure container. The differential transformer makes it possible to determine the position of a small iron cylinder affixed to the sample. Compressibility of vitreous silica determined by this method was obtained as $-\Delta V/V = p(26.43 - 0.0049t) \times 10^{-7} + p^2(21.8 - 0.040t) \times 10^{-12}$; compressibility is expressed in relative volume change, p in kg/cm², and t in °C.

From authors' summary

2614. **Lissitsyn, V. D., Microhardness measurements as a means of determining the degree of plastic change of form in metals** (in Russian), *Zavod. Lab.* **20,** 4, 474-478, 1954; *Ref. Zb. Mekh.* no. 11, 1956, Rev. 7970.

A method is developed of determining the degree of plastic change of form of metals by measuring their hardness.

From the results of tests in simple tension and corresponding hardness measurements, a chart has been plotted of the dependence of hardness on the quantitative characteristic of the intensity of deformation. With the help of this chart it is possible to obtain a numerical expression for the plastic change of form. This method, suggested by the abstractor and V. M. Rosenberg ["Technological problems of the theory of plasticity," Lenizdat, 1950] and presented by the author without indication of the source (the author substitutes microhardness for Vickers hardness, apparently without seriously prejudicing accuracy), is used for determining the degree of plastic deformation of a cupped, sheet-metal sample and the end fall of a rod, drawn between two rollers.

G. A. Smirnov-Alyaev

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2615. **Serabian, S., and Lockyer, G. E., Ultrasonic testing of large rotor forgings... criteria for detecting ability,** ASME Ann. Meet., New York, N. Y., Dec. 1957. Pap. 57-A-279, 10 pp.

Paper discusses the factors which influence the selection of the most suitable ultrasonic techniques for use in the nondestructive testing of large rotor forgings. It deals with the problem both theoretically and experimentally. Testing techniques for the initial forging inspection and the finished rotor assembly, and recent developments in these phases, are also considered.

From authors' summary

2616. **van Ouwerkerk, L., The relative advantage and limitation of nondestructive testing methods,** *Nondestructive Testing* **15,** 5, 298-312, Sept./Oct. 1957.

2617. **De Bruin, W., Surface roughness of large gear wheels for ship's turbines,** *Ingenieur* **69,** 33, 115-118, Aug. 1957.

Article deals with the application of an interference microscope with immersion cuvette for measuring the surface roughness of tooth flanks of large gear wheels.

From author's summary

2618. **Lubahn, J. D., On the applicability of notch tensile test data to strength criteria in engineering design,** *Trans. ASME* **78,** 1, 111-115, Jan. 1957.

2619. **Penkov, A. M., and Vorobeikov, A. M., Fatigue machine for testing steel cable wires with complex loading** (in Russian), *Zavod. Lab.* **21,** 7, 860-862, 1955; *Ref. Zb. Mekh.* 1956, Rev. 5556.

Description of a machine for fatigue tests of a wire of diameter between 0.3 and 3.0 mm for combined loads by tension, bending, and torsion, in which the tensile stress is constant, the bending over the arc of the circle of given radius is sign-determined, and the torsion for the given angle is also variable. The frequency of the cycles is up to 500 a minute.

S. I. Matveev

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2620. **Serensen, S. V., and Kozlov, L. A., High-frequency bending machine with programmed loading** (in Russian), *Vestn. mashinostroeniya* no. 4, 16-18, 1953; *Ref. Zb. Mekh.* 1956, Rev. 5621.

A substantiation of the necessity for studying the cyclic strength of metals for creating a variable stressed state in a component in accordance with its actual working conditions.

A description is given of the principle of operation and design of a machine (developed by L. A. Kozlov) for bending tests of 10-mm specimens, which are rotated at 12,000 rpm by a high-frequency electric motor, and at the same time subjected to the action of a variable load, the amplitude of which varies according to a specific law.

The machine may be equipped with a heating furnace and an appliance for creating an aggressive medium.

M. Ya. Shashin

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2621. Taylor, J., Research equipment for investigation of aircraft structure at elevated temperatures, *Aircr. Engng.* **24,** 342, 228–231, Aug. 1957.

Mechanical Properties of Specific Materials

(See also Revs. 2501, 2550, 2563, 2571, 2589, 2593, 2595, 2603, 2604, 2605, 2609, 2612, 2639, 2644, 2771)

Book—2622. Garside, J. E., Process and physical metallurgy, 2nd rev. ed., London, Charles Griffin & Co., Ltd., 1957, xx + 593 pp. 54s.

This comprehensive, well-written elementary introduction to the practice of process and physical metallurgy is appropriate as a first text to familiarize engineers with various aspects of this field. Viewed narrowly from a solid mechanics viewpoint, however, there appears to be little of interest.

G. Gerard, USA

2623. Walsh, J. M., Rice, M. H., McQueen, R. G., and Yarger, F. L., Shock-wave compressions of twenty-seven metals. Equations of state of metals, *Phys. Rev.* (2) **108,** 2, 196–216, Oct. 1957.

An explosive system is used to drive a strong shock wave into a plate of 24ST aluminum. This shock wave propagates through the 24ST aluminum into small test specimens which are in contact with the front surface of the plate. A photographic technique is used to measure velocities associated with the 24ST aluminum shock wave and with the shock wave in each specimen.

The measured velocities are transformed, using the conservation relations, to pressure-compression points. Resulting pressure-compression curves are given for 27 metals. The range of data is different for each material but typically covers the pressure interval 150 to 400 kilobars; probable errors in reported experimental pressure-compression curves are 1 or 2% in compression for a given pressure.

The experimental curves, which consist thermodynamically of a known P, V, E locus for each material, are used to calculate a more complete high-pressure equation of a state. This is done by means of a theoretical estimate of the volume variation of the Grüneisen ratio $\gamma(V) = V(\partial P/\partial E)v$. Calculated P, V, T states are listed for the various materials. For 24ST aluminum, quantities of application in shock-wave hydrodynamics are also tabulated.

From authors' summary

2624. Zaikov, M. A., The question of the flow curve when steel is deformed (in Russian), *Tr. Sibirsk. metallurg. in-ta* no. 2, 69–108, 1955; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8715.

On the basis of tests on the stretching of steel samples, and also on data given in the literature, consideration is given of the influence on the stress graph of the deformation, rate of deformation, temperature and composition of the steel. Tensile tests were made on samples of diameters of 6 and 10 mm, prepared from steels MSt_3, MSt_4, MSt_6, U8A, U10 and U12, on a 5-T press by Gagarin and on a 30-T hydraulic press, which was fitted with supplementary arrangements for changing the rate of deformation. The time and place of carrying out the tests is not stated. Tests were also made with samples of the self-same steels with a diameter of 14 mm, with various angular and cylindrical recesses. The speed of deformation varied stepwise in the range from 6.66×10^{-3} to 3.33 sec^{-1}, while the temperature varied from room temperature to 1150°. The dimensions of the deformed samples in the tests when the rates of deformation were high, and also the temperatures, were measured after the tests. The experimental data are published in a series of tables and graphs.

F. S. Churikov

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2625. Kovalev, K. V., Problem on the determination of deformation moduli for cast iron (in Russian), *Trudi Khar'kovsk. politekhn. in-ta* **5,** 2, 151–157, 1954; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8736.

The influences affecting the deformation of cast iron appear to be the rate of loading of the test sample, the form (mould), and dimensions of the sample. As the result of short-period applying and removing of the load, the cast iron acquires the properties of an elastic material, and also, when the furthest increase in load is applied, the cast iron again gives a residual deformation. The conditionality is established of the deformation characteristics obtained in the experiment. The necessity was pointed out of establishing a standard method for experiments for the determination of qualitative indicators of deformation, required in evaluating cast iron structures for rigidity, endurance and strength under vibration. Formulas are given for real and relative moduli of elastic and full deformation.

A method is explained for testing standard samples made of globular-graphite iron, unalloyed and modified with magnesium, for elongation and torsion.

The measurement of longitudinal deformation when elongation is in process and of the torsional angles when torsion is in process is carried out by means of a mirror extensometer. Curves are given for full and elastic deformation, for real and relative moduli of longitudinal deformation and shear-strains for cast irons of various brands.

A. F. Rozhnyatovskii

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2626. Probst, H. B., May, C. E., and McHenry, H. T., Corrosion resistance of nickel alloys in molten sodium hydroxide, *NACA TN* 4157, 26 pp., Jan. 1958.

2627. Rowe, J. P., and Freeman, J. W., Effect of overheating on creep-rupture properties of S-816 alloy at 1,500° F, *NACA TN* 4081, 75 pp., Dec. 1957.

2628. Opie, B. P., An investigation into the behaviour and influence of welded bracketed connections in aluminum alloy structural members, *Quart. Trans. Instn. nav. Arch.* **99,** 2, 204–225, Apr. 1957.

This paper is based on research work at the Department of Naval Architecture at Kings College, Newcastle-upon-Thyne, dealing with the influence of brackets on fixed beams made of aluminum alloy. Twenty six light-alloy specimens with various brackets were tested and the results analyzed. Strain-gage rosettes were used and the results were investigated according to Robson's method based on Mohr's strain circle. This method proved satisfactory, but no reliable solution has been found for the evaluation of bracket stresses. Bracketed connections are capable of obtaining a high degree of joint rigidity, but also bracketless connections of butt-welded members, suitably reinforced, seem to provide effective structural joints, as concluded from two separate tests. Also the lateral instability of brackets, twisting of unsymmetrical sections, end thrust on the specimens, degree of constraint due to brackets, twisting of unsymmetrical sections and the effect of bowing of brackets are investigated.

P. W. Abeles, England

2629. Stafford, J. A., and Gemmill, M. G., Stress-relaxation behaviour of chromium-molybdenum and chromium-molybdenum-vanadium bolting materials, Instn. mech. Engrs., Prepr., 8-16, 1957.

2630. Kingery, W. D., Klein, J. D., and McQuarrie, M. C., Development of ceramic insulating material for high-temperature use, ASME-AIChE Heat Trans. Conf., University Park, Pa., Aug. 1957. Pap. 57-HT-15, 6 pp.

Factors affecting the thermal conductivity of ceramics in the temperature range from 2200-4000 F are considered from the point of view of high-temperature insulation. The effective conductivity of solids and pores over a wide range of temperatures is analyzed. Particular attention is devoted to the effects of structure and phase distribution. This analysis is related to the development of optimum properties in thermal insulation for the temperature range considered. From authors' summary

2631. Youngs, R. L., Supplement to effects of tensile preloading and water immersion on flexural properties of a polyester laminate, *U. S. Dept. Agric., For. Prod. Lab. Rep.* 1856-A, 5 pp., June 1957.

2632. Youngs, R. L., Poisson's ratios for glass-fabric-base plastic laminates, *U. S. Dept. Agric., For. Prod. Lab. Rep.* 1860, 9 pp. + 2 tables + 1 fig., Jan. 1957.

Poisson's ratios were determined by test for several glass-fabric-base plastic laminates after normal conditioning. Loads were applied parallel and perpendicular to the warp direction to obtain Poisson's ratios along the natural axes of the materials. A few tests were also made to determine the effects of wet conditioning and type of resin on Poisson's ratios.

For most of the laminates investigated, Poisson's ratios were about 1/8. From author's summary

2633. Kocataskin, F., Vapour permeability of concrete (in English), *Bül. Istanbul tekn. Univ.* **10**, 1, 53-58, 1957.

The problem of vapor permeability of building materials is discussed. Present day theories on moisture movement through porous materials are briefly mentioned, the dry-cup (dessicant) method of determining vapor flow through building materials is described and a new graphical differentiation method is applied to evaluate test results obtained on two mortar samples of different mix proportions. A practical example is given to show the precalculation of the rate of vapor flow through a mortar coating of given qualities. From author's summary

2634. Ratner, S. B., and Sokol'skaya, V. D., Influence of the composition of a rubber on its static friction when slip is present (in Russian), *Khim. prom-st.* no. 1, 27-34, 1955; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8745.

Experimental data are presented to show the dependence of the frictional coefficient of a rubber on different factors, the hardness of the rubber, the fillers, the plasticizers, the degree of vulcanization, the type of rubber (latex), the character of the backing. Some general concepts of the nature of friction are also given.

E. N. Miroslavlev
Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

2635. Frenzel, W., and Banke, K.-H., The measurement of the curling tendency of yarns and twists (in German), *Faserforsch. u. Textiltech.* **8**, 4, 129-138, Apr. 1957.

The curling tendency of yarns and twists may lead to difficulties of production in twist and wearing mills in the hosiery and knitting industries when the physical properties of the fiber and the corresponding technological conditions during the production of the spun yarns are not taken into sufficient consideration. As criterion for the curling tendency of yarns and twists may be considered the turning moment acting crosswise to their longitudinal axis. The measurement of this turning moment with the aid of a new testing method forms the fundamental judgment of the curling tendency of yarns and twists.

From authors' summary

Plasticity, Forming and Cutting

(See also Revs. 2779, 2780, 2805, 2906)

2636. Takeyama, H., and Usui, E., The effect of tool-chip contact area in metal machining, ASME Ann. Meet., New York, N. Y., Dec. 1957. Pap. 57-A-45, 5 pp.

Cutting tests show frictional force on rake face proportional to tool-chip contact area. Normal force has two parts: (1) force due to normal stress, (2) force at cutting edge, a function of area of cut and rake angle. Ratio of contact area to area of cut is closely related to shear angle. R. N. Arnold, Scotland

2637. Nakayama, K., Temperature rise of workpiece during metal cutting, *Bull. Fac. Engng., Yokohama nat. Univ.* **5**, 1-10, Mar. 1956.

Paper concerns the determination of heat conducted into workpiece during orthogonal cutting by measuring the surface temperature rise. Formulated from the usual theory of moving heat source, author presents an expression which relates the heat conducted into workpiece per unit cutting area, q'_w, and the temperature rise at some designated location on the generated surface. It is found that $q'_w \sim V^0 t^0 \longrightarrow V^{-1} t^0$ as the thermal number of cutting increases, V being the cutting speed and t the feed. q'_w calculated in this way was then used to evaluate the "partition" constant λ which is the fraction of the shear zone heat flowing into the workpiece. When results were compared with those predicted from several existing theories on shear zone heat division, author concluded that Hahn's expression was among the best.

Reviewer noticed that the author was probably unaware of the excellent analysis made by Weiner [*Trans. ASME* **77**, 1955] on shear plane temperature distribution. There is also an apparent inconsistency in the application of Eq. [1] which is valid only for quasi-steady temperature rise, while Fig. (3) of the paper indicates that it cannot be the case. The mechanics described for the formulation of Eq. [2] is open to question.

B. T. Chao, USA

2638. Lyubimov, B. G., Methods for designing the blade profiles of a turbine drill (in Russian), *Neft. Kh-vo* no. 12, 12-17, 1955; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7473.

In designing the blade profiles of a turbine drilling rig, author has made use of the method of designing turbine blade profiles developed by M. J. Zhukovsky [*Ref. Zh. Mekh.* 1955, Rev. 1823].

Test results are given for the design of a fire-stage turbine for a drill with a diameter of 10 in. (25.4 cm), obtaining an efficiency of 0.725 for a flow of 50 litres/sec of water and a pressure drop of 2 atm (power output 9.73 hp). G. L. Grodzovskii

Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

2639. Rea, R. F., and Ripple, J. W., Cutting and grinding ceramics, *Bull. Amer. ceram. Soc.* **36**, 5, 163-167, May 1957.

A rapidly growing field is the precision grinding of fired ceramic products to very close tolerances. Due to dimensional changes in drying and firing, it is generally impossible to expect to hold tolerances closer than ±1%. With modern grinding facilities and techniques, very close finish tolerances on ceramics may be obtained, frequently ±0.0001 or better. Grinding also provides

improved surface finishes, the quality of which depends on the abrasive used, the body being ground, and grinding practices. The selection of proper abrasive wheels and the more important factors contributing to successful results are discussed.

From authors' summary

2640. Okushima, K., and Hitomi, K., On the cutting mechanism of soft metals, Proc. Sixth Japan nat. Congr. appl. Mech., Univ. of Kyoto, Japan, Oct. 1956, 187–192.

Orthogonal dry cutting of soft metals, such as lead and Lipowitz alloy, in low velocity, leads to the following conclusions:

(a) A flow region exists between the rigid region of workpiece and the plastic region of steady chip.

(b) The flow region associated with simple continuous chip varies in shape and size according to cutting condition, work material, etc. Sometimes it changes periodically during cutting.

(c) The cutting speed is one of the most important factors affecting the size of flow region. The higher the cutting velocity, the sharper the slip in the flow region.

(d) In the case of discontinuous chip, a flow region exists during formation of one chip fragment. The same is true with quasi-continuous chip. Fracture or crack occurs near the end boundary in the flow region, and is convex upward.

(e) Grid deformation under machined surface cannot be neglected. Grid line perpendicular to the uncut surface bends exponentially.

(f) Chip curls only for the action of stress distribution, even if there is no temperature distribution in shear zone.

From authors' summary

2641. Colwell, L. V., A method for studying the behavior of cutting fluids in wear of tool materials, ASME Ann. Meet., New York, N. Y., Dec. 1957. Pap. 57-A-160, 5 pp.

Refinements of design and instrumentation in a new friction and wear machine result in some new types of information which show promise of extending knowledge of the mechanisms of friction, lubrication, and wear. Provision has been made for varying normal load linearly with time and oscillating it between preset values. There is continuous recording of both the normal load and the friction reaction. Continuous recording at oscillating normal loads has discriminated sharply between lubricant additives. Close control of test conditions in wear studies has made it possible to identify high residual compressive stresses produced in finish grinding of cutting tools. From author's summary

2642. Brewer, R. C., On the economics of the basic turning operation, ASME Ann. Meet., New York, N. Y., Dec. 1957. Pap. 57-A-58, 7 pp.

2643. Johnson, W., and Woo, D. M., The pressure for indenting material resting on a rough foundation, *J. appl. Mech.* **25**, 1, 64–66, Mar. 1958.

A general expression is proposed for the indenting pressure of a flat rectangular punch producing a condition of plane strain. For the case of extreme pressure, computed values of the ratio of pressure to yield stress in shear range from 3.57 to 5.14 as the ratios of thickness of workpiece to width of indenter range from 0.71 to 1.735. No comparison with experimental results are presented. W. Schroeder, USA

2644. Ham, I., Roubik, J. R., and Bunce, J. P., Machinability of high-strength gray cast irons, ASME Ann. Meet., New York, N. Y., Dec. 1957. Pap. 57-A-239, 12 pp.

An analysis of the machinability of high-strength gray cast irons, similar to ASTM specification A 48-48 Class-40, from nine midwestern foundries with a general-purpose, cast-iron-cutting grade of carbide is given. Tool force, energy and power variations for the several cast irons over a range of speeds and feeds are depicted graphically. Individual and average tool-life values and curves obtained in tool-life tests as well as types of wear encountered are presented. For shop application, a general production-rate tool-life chart for turning this class of cast iron at a fixed depth of cut is shown. Since examination of these cast irons showed no great differences in microstructure, an attempt has been made to arrive at an empirical correlation between chemical composition and machinability in terms of tool life.

From authors' summary

2645. Stalker, K. W., Roll forming—chipless production, ASME Ann. Meet., New York, N. Y., Dec. 1957. Pap. 57-A-271, 9 pp.

There are three basic methods of producing mechanical products—cutting, fastening, and plastic forming. The process of working metal by plasticity is becoming increasingly appealing to the metal-working industry because of the rapid increase in material cost. This is also important because the cutting process costs money to remove the metal which is sold as scrap. The cold-forming process, as described in the paper, is primarily used for making cylindrical and conical parts in a wide range of metals. The mechanism of the process and the interrelationship of the processing variables are described. Some of these variables are speed, feed, material, and roll geometry. Examples of some of the products are shown which indicate the process has greater versatility than originally described by the sine law.

From author's summary

2646. Draper, A. B., Evaluation of tests for forgeability, *ASTM Bull.* no. 223, 62–68, July 1957.

Many laboratory forgeability tests are used, but there is little correlation among them and, at present, no standard forgeability test that can be easily performed and evaluated under shop conditions. From author's summary

2647. Kudo, H., A method for direct and continuous observation of internal metal flow during forming (in English), *Rep. Inst. Sci. Technol. Tokyo* **11**, 12, 147–154, 1957.

An apparatus for photographing the grid lines marked on a section plane of billet half through a glass plate was set up. Using this, the flow patterns of lead billets under forming of several types were photographed directly and continuously, and the method was confirmed to be effective for the study of metal flow. From author's summary

2648. Dodeja, L. C., and Johnson, W., The cold extrusion of circular rods through square multiple hole dies, *J. Mech. Phys. Solids* **5**, 4, 281–295, 1957.

Experiments were carried out to determine the pressure necessary to cold extrude pure lead, tellurium lead, pure tin and super-pure aluminum through 90° dies containing up to four holes arranged in different patterns. It is shown that, with the aid of a stress-strain curve for each material and knowing the reduction and lubrication conditions, this pressure is predictable to a fair degree of accuracy by assuming a simple expression for extrusion through a single hole die and by utilizing some of the empirical knowledge gained from the experiments described.

The paper concludes with a summary of the observations on the nature of the flow of the metal during its passage through the holes of the die. Some experimental results affording a comparison between plane-strain and axisymmetric pressures and flow patterns are also presented. From authors' summary

2649. Ananov, G. D., The kinematics of the three-dimensional mechanism of a grinding and polishing machine (in Russian), *Sb. statey Leningr. in-ta tochnoy mekhan. i optiki* no. 11, 78–86, 1954; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8091.

A graphical method is suggested for analyzing the kinematics of the three-dimensional mechanism of a grinding and polishing machine for spherical optical surfaces, embodying six linkages.

V. N. Geminov

Courtesy Referativnyi Zburnal, USSR

Translation, courtesy Ministry of Supply, England

Hydraulics; Cavitation; Transport

(See also Revs. 2474, 2665, 2666, 2667, 2676, 2686, 2803, 2871, 2877)

Book—2650. Arghyropoulos, P., Calculation of the flow in closed channels and in channels with free surface [Calcul de l'écoulement en conduites sous pression ou à surface libre], Paris, Dunod, 1957, xiii + 326 pp.

Book contains tables for computation of flow in circular and ovoidal, full or partly filled, conduits and in open channels, for use of hydraulic engineers. Direct solution is given for conduits of 50 to 2000-mm diameter and slopes 0.0001 to 0.5. Manning formula is applied with roughness coefficient $n = 1/77 = 0.013$; corrections for other values are necessary. Semigraphical tables for partly filled conduits are similar to those by E. Wild and O. Schoberlein (1931, 1952). Computation of open channels is less convenient: they are related to closed conduits. Table of roughness coefficients by R. E. Horton (1916) is given, disregarding better collections presented in past 40 years. Unnecessary table for conversion into abandoned Chézy formula is given; three nomograms are included. Strickler formula is mentioned in the title only.

Print is good, except for erroneous page references throughout the text. Tables are carefully prepared in metric units. The Manning formula tables by H. W. King and hydraulic and excavation tables of the Bureau of Reclamation are more convenient for open channels, and are in English units. S. Kolupaila, USA

2651. Stephenson, D. G., Fluid friction in partially filled circular conduits, *Trans. Engng. Inst. Canada* no. 1, 39–43, Sept. 1957.

The pressure loss due to friction was measured for conduits ranging from ¾ in. to 1¼ in. nominal diameters containing up to three insulated copper wires. The air flow rates used gave Reynolds numbers ranging from 5000 to 50,000. It was found that all the results could be correlated in terms of an equivalent diameter, i.e., the diameter of an empty conduit which would have the same pressure-loss mass-flow characteristics as the conduit containing wire. The ratio of equivalent diameter to conduit diameter was found to depend primarily on the ratio of wire diameter to conduit diameter and the number of wires in the conduit, and only slightly on the Reynolds number. From author's summary

2652. Wadlin, K. L., and Christopher, K. W., A method for calculation of hydrodynamic lift for submerged and planing rectangular lifting surfaces, *NACA TN* 4168, 34 pp., Jan. 1958.

A method for the calculation of hydrodynamic lift of submerged and planing surfaces having aspect ratios varying from 0.125 to 10 and dihedral angles up to 30° has been presented. Results appear to agree with experimental data. Y. V. G. Acharya, India

2653. Parkin, B. R., and Peebles, G. H., Calculation of hydrofoil sections for prescribed pressure distributions, Soc. nav. Arch. mar. Engrs., Prepr. no. 1–17, 55 pp., Dec. 1956.

Author recalls the theories of incompressible, inviscid, steady flow and conformal transformations, and deduces a kind of manual for the design of hydrofoils and airfoils from the specification on the pressure distributions. A profile design is begun by tentatively specifying the pressure desired at selected angles of attack. These specifications, which usually conflict with each other, are reconciled so as to correspond to an existing but unknown profile. The coordinates of the profile are then calculated.

L. J. Tison, Belgium

2654. Ransford, G. D., Aerodynamic guides used in novel forebay design (in French), *Houille blanche* **10**, 3, 430–448, July 1955.

By placing guides of aerodynamic shape in a head-race canal, it was possible to prevent a breakaway of flow in the curved, diverging section at the entrance to the forebay. The location of the guides was determined by scale-model investigations; on the basis of the results obtained, and availing himself of certain formulas derived by other investigators, author outlines an approximate method of analysis which may prove useful in future studies of this type. From author's summary

2655. Moran, P. A. P., The statistical treatment of flood flows, *Trans. Amer. geophys. Un.* **38**, 4, 519–523, Aug. 1957.

The estimation of return periods of floods is considered from the point of view of mathematical statistics. This requires, essentially, the estimation of the tail of a probability distribution from a sample of values which is usually not dense in this tail. The error in the estimation of a flood corresponding to a given probability arises from two sources: (1) the uncertainty as to the mathematical form of the distribution; (2) the uncertainty arising from the statistical errors of estimation of the parameters of the distribution which occur because of the finiteness of the length of record. These errors are illustrated on a numerical example by fitting a log-normal distribution and a type III distribution to fifty annual values of extreme monthly flow of the River Murray.

From author's summary

2656. Proudman, J., Oscillations of tide and surge in an estuary of finite length, *J. Fluid Mech.* **2**, 4, 371–382, June 1957.

Author considers an estuary of uniform cross section, open to the sea at one end and closed at the other and supposes that an incident long wave of given general form enters from the sea; this wave represents a combination of tide and surge as generated in the sea. The equations of continuity and motion reduced to their linear terms give the reflection of this wave at the head of the estuary. Author presents the next approximation when the nonlinear terms are retained, and it appears that the surface elevation at the head of the estuary depends on the primary elevation there at the same time and for a previous interval during which a progressive wave could travel twice the length of the estuary. When, during this interval, the primary wave increases steadily to a maximum, the effect of the shallow-water terms of the differential equations is to make high water higher and earlier, while the influence of the frictional term is to make high water lower and later.

Author also discusses the problem of a short estuary and the separation and interaction of tide and surge.

L. J. Tison, Belgium

2657. Lencastre, A., Theoretical and experimental design of shaft spillways (in Portugese), *Minist. Obras Publ., Lab. Engen. civ. Lisboa Pub.* no. 88, 16 pp., 1956.

A general review of the problem of designing a shaft spillway is first presented: design of the crest, transition between crest and shaft, design of shaft, shape of the bend, need of aeration, design of the diversion tunnel, and study of the get-away conditions downstream.

Results of model tests carried out in the "Laboratório Nacional de Engenharia Civil" are given. "Fluvial Hydraulics Section" on the spillways of Campilhas, Silves, Maranhão and Montargil schemes are summed up.

The conclusions drawn from some systematic laboratory researches on shaft spillways are presented.

From author's summary

2658. Lebedev, I. V., The problem of local erosion behind a horizontal barrier (in Russian), *Gidrotekhn. str-vo* no. 8, 40–43, 1954; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8260.

Criticisms are presented to the method of reciprocal depths developed by M. S. Vyzgo for calculating local erosion in channels behind a horizontal barrier. It is pointed out that Vyzgo used an incorrect method for determining the coefficient k_1 in the formula for depth of erosion; this causes wide divergence from the experimental values obtained by D. I. Kumin [*Izv. Vses. n. -i. in-ta gidrotekhn.* 40, 1949; 47, 1951]. The author refutes the fundamental proposition of Vyzgo that the depth at the point of maximum erosion is proportional to the second reciprocal depth, and objects to extension of the relationship obtained for turbulent conditions to the case of erosion by a steady flow. I. I. Levi

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2659. River damming by "Hydraulicking" (in Russian), *Byul. nauch.-tekhn. inform. po melior. i gidrotekhn.* no. 1, 37 pp., 1955; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8267.

It is suggested by the All-Soviet Scientific Research Institute for Hydraulic Engineering and Land Drainage to dam streams by using a combination of "suspended" and "transportable" soil particles, deposition taking place as a result of overloading the flow beyond its transporting capacity.

The method of calculation suggested contemplates only suspended and transportable deposits, and is inapplicable to non-transportable sedimentation.

The attempts to extend the calculation technique to non-transportable gravel and pebble fractions, which, for instance, form the deposited dam in the barring of the river Dniester in the construction of the hydroelectric power station at Dubosary, have proved mistaken, since this distorts the physical significance of the process of alluvial deposition. The tests made of the process of hydraulicking in the construction of the Kakhovka hydroelectric power station in 1955 have shown the inapplicability of this method for heads exceeding 0.5 m, as well as the incompleteness of the methods of calculating the end face construction and disperse deposition. M. A. Dement'ev

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2660. Surova, N. N., A spreader turnout of radial planform (in Russian), *Gidrotekhn. str-vo.* no. 7, 38–39, 1955; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8255.

The results of laboratory tests are presented on a fanning turnout with a dentated apron sill at the end, intended to create favorable conditions for leading the flow of a spillway into a diversion channel. The spillway is of trapezoidal section and calculated for flow volumes between 17.5 and 60 meters^3/second, with a difference in levels of the structure of 50 meters.

As a result of the investigations, performed on a model to a scale of 1:35, a variant of a fanning turnout of simplified construction has been developed, in which the horizontal spread of the flow has been increased from 4 meters to 19.6 meters, at the end of a spillway of 35 meters length. The dentated apron sill of the suggested design enables the flow from the structure, with the flow volumes indicated above, to be discharged at 25 + 50 meters, the depth of erosion of the channel at the point of fall being 4 ÷ 7 meters. M. F. Skladnev

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2661. Malcor, R., Equilibrium of limiting capillaries: theory of capillary extraction *(in French), Ann. Ponts. Chauss.* **127**, 4, 473–500, July-Aug. 1957.

Problem of centrifugal action on fluids in a porous bed is approached by considering the menisci formed by liquid held between two tangent spheres. Condition for stable menisci is established theoretically and verified experimentally, permitting limit menisci to be related to a single dimensionless parameter involving specific mass of fluid, acceleration, radius of curvature and capillary constant. Study allows determination of fluid volume retained for specific centrifugal action. Results apply to behavior of moisture in soils and liquid - liquid displacement in bed of spherical particles. W. E. Ibele, USA

2662. Prostak, F. A., Some particular features of the methodology and technique of making cavitation tests on hydraulic turbine models (in Russian), Hydraulic Turbine Construction no. 1, Moscow-Leningrad, Mashgiz, 1955, 81–91; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8289.

Different methods are examined of determining the appearance of cavitation in hydraulic turbines, in laboratory model tests. The object of the paper is to elucidate the different methods, in order to determine their suitability for use in the case of full-scale turbines. No conclusive recommendations are made.

The following methods are discussed: vibrational, acoustic, etching of the exposed surfaces, variation of the external characteristics of the turbine. The greatest attention is devoted to the external characteristics of the turbine. Diagrams of cavitation test stands are given, and methods of experimentation described; for evaluation of the results and determination of the practical significance of the coefficient σ, it is recommended to use the relationships $n_1'(\sigma)$ and $M_1'(\sigma)$ instead of $\eta(\sigma)$. K. K. Shal'nev

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2663. Rukin, S. P., and Stopsky, S. B, Stroboscopic observation in cavitation tests on turbine models (in Russian), Gidroturbostroyeniye no. 1, Moscow-Leningrad, Mashgiz, 1955, 92–103; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8290.

An electrical stroboscope layout is described, with an installation for pulse-recording cavitation in turbine rotor models. The results are communicated of some tests using the above apparatus, adapted to particular points in the cavitation curve $\eta(\sigma)$.

Depending on the blade-setting angle, number of revolutions of the turbine, the cavitation number σ, geometrical form of the blades and hub, the following cavitation forms were identified: slot cavitation at the casing, slot cavitation at the hub, profile cavitation on the blades. These forms of cavitation are encountered separately and in all possible combinations. Cavitation at the casing usually appears before the other forms, and at higher values of σ. At the critical value of σ corresponding to a falling value of η, cavitation already assumes fully-developed forms. It is necessary to considerably increase the σ value of the installation in order that the turbine shall work outside the cavitation range.

Shortened hub fairings cause the cavitational properties of the turbine to deteriorate, outside the limits of the combined working condition of the turbine. K. K. Shal'nev

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

Incompressible Flow: Laminar; Viscous

(See also Revs. 2409, 2472, 2582, 2652, 2653, 2654, 2727, 2738, 2739, 2742, 2743, 2803, 2804, 2809, 2871, 2886)

Book—2664. Landshoff, R. K. M. (edited by), Magnetohydrodynamics, Stanford, California, Stanford University Press, 1957, x + 115 pp. $4.

This compilation of the papers presented at the Magnetohydrodynamics Symposium held on December 29, 1956 is divided into two sections, namely "Theoretical work" and "Laboratory experiments".

A. R. Kantrowitz and H. E. Petschek present an introductory discussion of magnetohydrodynamics. They classify the domains of gaseous magnetohydrodynamics and define the various terms pertinent to the field. W. M. Elsasser discusses the dimensional aspects and presents the fundamental equations. He discusses the creation of magnetic loops. F. Hoyle in his paper concerns himself with the properties of completely ionized gases with particular reference to the build up of large magnetic fields inside stars. J. M. Burgers discusses the application of the fundamental equations to the case when a plane shock wave penetrates into a magnetic field. Mr. Rosenbluth considers the dynamics of a pinched gas. In the second section, R. K. M. Landshoff presents scaling laws which may be used in the design of experiments. Magnetically driven shock waves are discussed by A. C. Kolb, S. W. Kash and others. An experiment on liquid sodium instability is described, and W. A. Newcomb presents a treatment of the hydromagnetic wave guide proposed by R. F. Post.

This little volume concerning the many aspects of magnetohydrodynamics contains multifarious information. It should be invaluable to all persons working or interested in this field.

A. B. Cambel, USA

Book—2665. Birkhoff, G., and Zarantonello, E. H., Jets, wakes, and cavities, New York, Academic Press, Inc. (Applied mathematics and mechanics. Vol. II), 1957, xii + 353 pp., $10.

In this very remarkable work the authors give a critical and, wherever possible, unified presentation of ideas, methods, and results obtained by three generations of mathematicians, physicists, and engineers in the hydrodynamics of free-streamline flow, including, of course, the important contributions of the authors themselves. The scope is not restricted to perfect-fluid theory, but, naturally, plane and axisymmetric potential flow take up three fourths of the book. A bibliography of almost one hundred basic references is given, but the number of papers quoted in footnotes (and, unfortunately, not collected in an author's index) must be very much larger. Plane flow theory is presented in chaps. II–VII; chaps. X and XI are devoted to some problems of axisymmetric and unsteady flow; viscous flow in wakes and jets is the subject of chaps. XII (laminar) and XIV (turbulent), while chap. XIII deals mainly with the vortex-street model of the wake. Compressible flow with free streamlines (mainly subsonic Chaplygin jets and generalizations) occurs in chap. VIII which, somewhat unexpectedly, ends with a brief discussion of potential flows with gravity. Besides some tabulated results, there are ten plates of flow nets for Helmholtz flow past plates at varying incidence, of jet deflections, and of Riabouchinsky flow. An interesting discussion of the numerical techniques here employed is found in chap. IX, which retains much of its validity although computing machinery and coding techniques have changed in the meantime. The last chapter, XV, is a collection of recent experimental results in cavitation, entry of a solid into water, bubble dynamics, etc., touching in one way or another on the principal topic of the work.

A completely systematic account, obviously, is possible only when a fully developed analytic theory exists. Plane flow is here the case in point, and the authors have indeed brought order into the maze of results accumulated in a hundred years of applying conformal mapping to free-solid boundary problems. Order is achieved by the concept of "simple flow": a simply connected open domain R of the flow plane z, carrying a locally (but not necessarily in the large) single-valued complex velocity $\zeta(z)$, analytic in R and bounded on the boundary of R, which consists of a finite number of rectifiable streamlines. Such a domain can be mapped one–one on a half-plane $\mathrm{Im}\,(T) > 0$; T serves as parameter for the representation of ζ and of the potential W.

The principal result is that the singularities of $W(T)$ can be only poles of not more than second order and logarithmic branch points at real T-values and at $T = \infty$. This leads to canonic forms for $W(T)$ and dW/dT, and not only permits a systematic survey of such classical examples as Kirchhoff and Réthy flows, but also a topological classification of the possible streamline configurations in simple flows in chap. III. In chap. IV a more general simple-flow concept (connectivity only locally simple) is used, and again classification of the singularities and of the asymptotic behavior of the streamlines is obtained. General theorems on the differential geometry of streamlines (e.g. curvature at separation) and an account of recently discovered comparison theorems conclude this chapter, the results of which are applied in chap. V, dealing with multiple plate problems such as impinging jets from nozzles, Riabouchinsky flows, cusped cavities, and also with interior sources and vortices. Chapter VI begins with Levi-Civita's method for flow past curved barriers (here within a jet) with pertinent examples, brings integral-equation formulations for the direct problem, and begins the discussion of the "parameter problem", which is taken up more completely in chap. VII devoted to existence and uniqueness. Here the concise and original account of Weinstein's function-space, and variational methods will please the mathematician.

Chapter X on axisymmetric problems, necessarily more fragmentary than the previous chapters, surveys briefly the results obtained by ring-shaped singularity distributions, justly criticizes the representation of cavity flows by Rankine bodies, and reviews approximate methods. Dynamics of gas-filled cavities, impact problems and the associated induced mass are treated in chap. XI, which ends with a somewhat hasty survey of Taylor and Helmholtz instability.

In the "Navie-Stokes" chap. XII, asymptotic wake behavior is analyzed in the (simplified) Oseen approximation, a simple and lucid account of the momentum theorem is applied to the same problem, and similarity properties of wake and jet profiles are tabulated. Boundary-layer approximation appears in connection with the wake velocity profile and with Schlichting's circular and plane jet. In this survey chapter a broader choice of topics might perhaps have been possible. Chapter XIII, on the other hand, is again very original in its critical representation of the various forms of the vortex-street wake theory, influenced by Professor Birkhoff's own contributions to this subject. One misses, however, a reference to Maue's work and to Coddington's reinvestigation of the classical stability problem. Chapter XIV reapplies the asymptotic theories of chap. XII to turbulent wakes. Prandtl's mixing-length idea takes some beating, although it (or some other transport hypothesis) is at present indispensable. This chapter relies largely on references.

More than once the authors discuss the limitations of the Euler-Helmholtz model and state that the Navier-Stokes flow model can only be successfully employed if the free streamlines are boundaries between fluid and gas or vapor, whereas it is completely useless as an approximation to wake behavior. While this is certainly so in general, there is a Reynolds number range where that model does not do so badly. And why not at the same time admit that the Euler-Helmholtz model fails also along solid boundaries, when boundary-layer effects take over?

By and large, however, this book is a very important contribution to the theory of fluid flow. It is bound to deeply influence research and advanced teaching and it is unique as a reference work. With the exception of chap. VII, the mathematical level of the book is not beyond the reach of the well-trained fluid dynamicist.

G. Kuerti, USA

Book—2666. Prandtl, L., and Tietjens, O. G., Applied hydro- and aeromechanics, New York, Dover Publications, Inc., 1957, xvi + 311 pp. $1.85. (Paperbound)

This book was originally written in 1934 and translated by J. P. den Hartog. The present Dover edition is an unabridged and unaltered reproduction of the first edition.

The editors

Book—2667. Prandtl, L., and Tietjens, O. G., Fundamentals of hydro- and aeromechanics, New York, Dover Publications, Inc., 1957, xiii + 270 pp. $1.85. (Paperbound)

This book was originally written in 1934 and translated by L. Rosenhead. The present Dover edition is an unabridged and unaltered reproduction of the first edition.

The editors

2668. Feng, T.-Y., Steady-state axial flow forces on pneumatic spool-type control valves, ASME Ann. Meet., New York, N. Y., Dec. 1957. Pap. 57-A-129, 5 pp.

The momentum theory is used to determine the steady-state axial-flow forces on a pneumatic control-valve spool, caused by the compressible fluid passing through the sharp-edged orifice. The experimental results compare favorably with the theoretical calculations for valve displacements of 0.005 in. and less at supply pressures not exceeding 500 psig. Pneumatic flow forces as compared with hydraulic flow forces on the same valve spool are higher in the unchoked flow regime [$(P_{down}/P_{up}) > 0.528$] but lower in the choked-flow regime [$(P_{down}/P_{up}) \leqslant 0.528$].

From author's summary by A. I. van de Vooren, Holland

2669. Zalmanzon, D. A., The self-oscillation of systems with pneumatic governors embodying closed capsules (in Russian), Trans. Second All-Soviet Congress on the theory of automatic control, Vol. 1, Moscow/Leningrad, Akad. Nauk SSSR, 1955, 266–298, *Ref. Zh. Mekh.* no. 12, Rev. 8064.

Curves are given enabling: (1) The steady-state pressure to be determined in any part of a branching, pneumatic system embodying capsules connected by calibrated orifices (nozzles) or ducts; (2) the numerical values to be determined of the coefficients of the differential equations of small oscillations of the system. These equations are supplemented by equations describing the pressure variations in closed capsules, which are nonlinear even in the presence of small oscillations. It is demonstrated that the nonlinear characteristics of closed capsules may lead to the appearance of self-oscillation. The parameters of the self-oscillation are found by the method of direct linearization according to Ya. G. Panovko [*Inzhen. Sbornik* **13**, 1952], or by the method of harmonic balance.

Curves of the referred transmission function of a closed capsule are given to facilitate application of the method. The admissibility of a series of simplifying assumptions in the calculation of self-oscillation is demonstrated.

A comparison is made of the results obtained by the method of Ya. G. Panovko and the method of harmonic balance, respectively.

M. A. Aizerman

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2670. Walker, J. E., Whan, G. A., and Rothfus, R. R., Fluid friction in noncircular ducts, *AIChE J.* **3**, 4, 484–489, Dec. 1957.

Pressure drop due to fluid friction has been measured in a smooth tube; in six smooth concentric annuli; and between smooth, parallel, flat plates. The data cover the viscous, transition, and lower turbulent ranges of flow. Friction factors are in agreement with theory in the viscous range and can be correlated uniquely in the fully turbulent range by means of a modified hydraulic radius concept. Limits of the transition region in annuli are functions of the ratio of inner and outer radii. Correlations in the fully turbulent range permit friction factors for the noncircular ducts to be predicted within the uncertainty of smooth-tube data. Such factors can be used for the purpose of correlating other variables as well as for direct calculation of pressure drop.

From authors' summary

2671. Happel, J., and Brenner, H., Viscous flow in multiparticle systems: Motion of spheres and a fluid in a cylindrical tube, *AIChE J.* **3**, 4, 506–513, Dec. 1957.

Paper analyzes free fall of a cloud of spheres through laminar flow in a vertical, round tube. The restrictions are (a) zero Reynolds number (Stokes flow) for the relative motion, (b) sphere diameter much less than tube diameter, (c) extreme dilution, i.e., no interaction between spheres.

No mention is made of the inevitable sphere rotation in a velocity gradient or of the side force which must result (with or without spin) when there is relative translation of sphere and fluid in a fluid velocity gradient.

Reviewer believes that this paper represents a very rough approximation indeed.

S. Corrsin, USA

2672. Kline, S. J., Moore, C. A., and Cochran, D. L., Wide-angle diffusers of high performance and diffuser flow mechanisms, *J. aero. Sci.* **24**, 6, 469–470 (Readers' Forum), June 1957.

2673. Riparbelli, C., A characteristic of stable flow of real fluids (in Italian), *Aerotecnica* **37**, 1, 13–23, Feb. 1957.

Paper deals with the application of variational principles to the flow of viscous fluids. After discussion of an analogy between deformation of an elastic incompressible body (gelatine) and the flow of an incompressible viscous fluid, author states: "The distribution of *displacements* in a body of gelatine subject to a given distribution of forces and pressures is numerically equal to the distribution of *velocities* in a viscous incompressible fluid under conditions of geometrical similarity and same boundary conditions and distribution of forces, provided that the time unit be chosen in such a way that μ be numerically equal to G" (μ viscosity; G shear modulus). A comparison of the Navier-Stokes equations with the equations for the equilibrium of an elastic body, however, shows that the acceleration terms of the former are not present in the latter, and, hence, that this analogy is not true. Therefore, conclusions based on the quoted statement are not valid.

Reviewer does not question the application of variational principles to viscous flow which, with their limitations, has been directly and clearly presented in previous publications on the subject [Millikan, C. B., *Phil. Mag.* April 1929; Lichtenstein, L., "Grundlagen der Hydromechanik," Berlin 1929; Villat, H., "Lecons sur les fluides visqueux," Paris, 1943. See also earlier work by Helmholtz, Korteweg, Rayleigh].

E. O. Macagno, Argentina

2674. Kramer, J. J., Prian, V. D., and Wu, C.-H., Theoretical analysis of incompressible flow through a radial-inlet centrifugal impeller at various weight flows, *NACA Rep.* 1279, 16 pp., 1956.

[See AMR **9**, Rev. 540]

2675. Taliyev, V. N., Ventilation aerodynamics (in Russian), Moscow, Stroyizdat, 1954, 288 pp. + illus.; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8182.

Book is a monograph on the aerodynamics of ventilation. Author examines a series of problems by the methods of theoretical aero-

dynamics, with the first part of the book briefly expounding the fundamental principles. A great number of examples of the solution of particular problems is given. The second part deals with the application of the methods of theoretical aerodynamics to a number of ventilation problems.

The third part gives brief information on the subject of experimental aerodynamics as applied to the problems of ventilation. The aerodynamic analysis of wind tunnels is described, experimental installations, measuring equipment and the methodology of experimental research are succinctly discussed.

Book is a useful aid for scientific workers and technicians in the field of heating and ventilation, as well as in other fields dealing with the motion of liquids and gases.

I. E. Idel'chik
Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

2676. Zhuravlev, P. A., The application of the method of Academician S. A. Kristianovich to the study of fluid motion in ducts (in Russian), *Vestn. Leningr.* in-ta no. 8, 67-85, 1955; *Ref. Zh. Mekh.* no. 12 1956, Rev. 8245.

For the solution of the approximate, nonlinear equations of motion of fluids in open channels (the conventional problem of the hydraulics of aqueducts), and applying the gas-hydraulic analogy indicated by N. E. Joukousky [*Trans. CAHI* 1925, 1], author uses the method of S. A. Kristianovich for the solution of the equations of gas dynamics at subsonic velocities [*Prikl. Mat. Mekh.* 1947, II]. In this method, the said equations are transformed into two systems of linear equations for an arbitrary, auxiliary flow.

The boundary conditions for the stream function and velocity potential become more complex, and the solution of the direct problem—determination of the motion of a fluid in a channel with specified boundaries and boundary conditions—becomes, in general, more difficult. It is, however, possible to solve the inverse problem, from a given plane-parallel, auxiliary flow, to determine the motion of the fluid in a particular channel.

One of the two systems obtained is solved for the plane of the auxiliary flow, approximately, on the assumption of a constant value of the coefficient of the system, which is valid with sufficient accuracy for the condition when the flow velocity differs little from a particular, mean velocity. With the help of the equations obtained, it is possible to solve the problem of the motion of a fluid along a section of the duct, converging in the direction of flow, the sides whereof are assumed everywhere vertical and parallel, both in the wide and the constricted parts of the channel.

The complex potential of the auxiliary flow is found in the form

$$W = U\zeta + \frac{m}{2\pi} \ln \cos \frac{\pi\zeta}{ai}$$

where

$$\zeta = \xi + i\eta; \quad -\infty < \xi < +\infty; \quad -\frac{1}{2}\alpha < \eta < +\frac{1}{2}\alpha$$

the parameters of this flow, U, m, a, and its flow volume, are selected according to the condition of least deviation of the characteristic values, determining the motion of the fluid in the channel, in the transition from the plane of the auxiliary flow, to the plane of the actual flow.

The data obtained are correlated with the experimental results obtained in the VNIIG laboratories. It is found that the values of the depths, measured in the symmetry plane of the launder, coincide within the limits of accuracy of the measurements with the values obtained by calculation. Over the width of the flow, however, the depth changes relatively little; hence the results of depth measurements across the width of the launder only qualitatively confirm the results by calculation.

Author does not mention the papers of G. I. Sukhomiel [Visti Institut gidrologii i gidrotekhniki Akad. Nauk URSU, 1948, Nos. 3, 4], in which equations are derived and solutions obtained for the problem of the motion of a fluid in an undisturbed flow, both by transformation according to S. A. Kristianovich, and by other methods.

G. I. Sukhomel
Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

2677. Craig, P. P., and Pellam, J. R., Observation of perfect potential flow in superfluid, *Phys. Rev.* (2) **108,** 5, 1109-1112, Dec. 1957.

The lift on an airfoil in a velocity field of perfect superfluid flow within liquid helium II has been found to vanish for sufficiently low velocity. Thus the flow is pure potential flow with no circulation, and the viscosity boundary condition of zero velocity at the trailing edge (Kutta condition) which is normally introduced does not apply. We believe this to be the first illustration of absolutely perfect hydrodynamic flow, i.e., one in which the viscosity vanishes identically and therefore asserts no influence on the flow field.

From authors' summary

2678. Tamada, K., and Fujikawa, H., The steady two-dimensional flow of viscous fluid at low Reynolds numbers passing through an infinite row of equal parallel circular cylinders, *Quart. J. Mech. appl. Math.* **10,** 4, 425-432, Nov. 1957.

By using Oseen's approximation ... "it has been found that the drag acting on any one of the cylinders is always greater than that acting on the same cylinder when it is immersed alone in an unlimited uniform flow of the same velocity. In particular when the Reynolds number is sufficiently small, the drag is found to be proportional to the flow speed U, ..."

The application of these results in the Reynolds range of less than unity to engineering problems will probably be restricted to instrument development where these results are particularly adapted to measurement of flow rates, viscosity, or velocity.

E. E. Covert, USA

2679. Murray, J. D., Non-uniform shear flow past cylinders, *Quart. J. Mech. appl. Math.* **10,** 4, 406-424, Nov. 1957.

A general method is described whereby an approximation of any desired degree of accuracy to the stream functions for two types of variable shear flow past finite cylinders can be obtained. The two shear distributions in the free stream can be approximated to the linear shear distribution and the shear present in an unretarded incompressible boundary layer. In every case the stagnation streamline is displaced from the position opposite the line of symmetry of the cylinder, and general expressions are obtained for this displacement. The line of symmetry may be in the direction of or perpendicular to the direction of flow.

R. C. Binder, USA

2680. Levine, H., Skin friction on a strip of finite width moving parallel to its length, *J. Fluid Mech.* **3,** 2, 145-158, Nov. 1957.

A flat strip of infinitesimal thickness, infinite length, and finite width is located in a viscous incompressible fluid; both the strip and the fluid are at rest initially. An expression for skin friction is derived for the case of the strip suddenly set into steady motion parallel to its length. The expression is applicable in the early stages of motion, and consists of a correction to existing solutions for strips of infinite or semi-infinite width.

From author's summary by G. M. Low, USA

2681. Bloomer, N. T., Note on the position of ring singularities in an axisymmetric potential field, *J. Fluid Mech.* **3,** 2, 217-220, Nov. 1957.

In an axisymmetric potential field, if $\phi = 2f(x)$ on the axis then it is possible to find ϕ everywhere in the field, as is well-known. In two-dimensional (2D) space with a line of symmetry, the value of ϕ is given by $2\phi = f(z) + f(\bar{z})$. Author finds the corresponding form for 3D and shows that singularities of the 2D function will give the points where the corresponding ring singularity in 3D cuts a plane through the axis of symmetry. A reduction of 1/2 in the order of singularity seems to be general in going from 2D to 3D. Author suggests that this theory can be applied to find singularities of velocity field in a nozzle.

J. C. Cooke, England

2682. Queijo, M. J., and Fletcher, H. S., Low-speed experimental investigation of the Magnus effect on various sections of a body of revolution with and without a propeller, *NACA TN* 4013, 9 pp. + 1 table + 26 figs., Aug. 1957.

An experimental investigation has been made at low speed to determine the Magnus effect on various sections of a body of revolution of which several sections could be rotated either as individual units or in combinations. The investigation included the measurement of the Magnus effect on the body alone and also on the body with a three-blade propeller. The tests were made over an angle-of-sideslip range from $-5°$ to $30°$, and at rotational speeds from 0 to 8000 rpm with the propeller off and from 0 to 5000 rpm with the propeller on.

The results of the investigation showed that with the propeller off, rotation of sections in the expanding portion of the body (that is, at the nose of the body) produced no appreciable Magnus force on these sections. However, rotation of these sections did produce an appreciable Magnus force on the part of the body downstream of the maximum diameter. These results are believed to be associated with the position and strength of the vortices shed from the body. With the propeller off, the center of pressure of the Magnus force on the body was about 70% of the body length behind the nose, and moved forward with increase in rotational speed. With a propeller on the body of revolution, a large Magnus force was developed on sections of the body behind the propeller. The Magnus force on sections ahead of the propeller was small and independent of whether these sections were rotating.

From authors' summary

2683. Huber, E. W., Contribution to the calculation of unsteady fluid flows with special regard to the charging processes in internal combustion engines, *VDI Forschungsheft* **23,** 462, 32 pp., 1957.

A method is given to calculate the process of changing charges in piston engines. The flow out of a pipeline through an independently driven rotating slide valve into a low-pressure reservoir is taken as a model. Assuming small amplitudes of pressure disturbances and using the partial differential equation of one-dimensional acoustic theory, some models with different lengths of the pipelines and different cross sections of the throttles are computed and the results are compared with those of the new theory which works with the unsteady flow of an incompressible gas (thus essentially taking into account the dynamical mass effects of the streaming gases only). Good agreement between the two methods is obtained for the time history of the flow velocity in the throttle section if the opening time of slide valve is sufficiently great compared with the transit time of the pressure waves in the pipeline. The process of inflow into the cylinder of a normal two-stroke or four-stroke internal combustion engine, for instance, falls into the range of validity of author's theory. The method is less suitable for the process of outflow from such engines, especially if there are long exhaust pipes, because in these cases the influence of heat exchange and of temperature in the exhaust gases plays a certain part and because entropy changes not taken into account here must be considered in a stepwise calculation of the propagation of the pressure waves.

It is clear that the method is not usable for great pressure amplitudes, as for instance in the case of resonance. But in its range of validity it is superior to earlier known methods [H. List "Die Verbrennungskraftmaschine," Vol. IV, part 1, Wien 1949], being both less laborious and more accurate. This increased accuracy is shown by some experiments of the author. Of course, the method may also be used within its given limits to calculate other processes in which gas columns are accelerated or retarded.

Working diagrams permit easy graphical evaluation of the variation with time of the flow velocity in the control mechanism and, thence, determination of the law of charging.

H. Behrbohm, Sweden

2684. Chisholm, D., and Laird, A. D. K., Two-phase flow in rough tubes, ASME Semiann. Meet., San Francisco, Calif., June 1957. Pap. 57-SA-11, 9 pp.

Paper presents data for pressure drop and saturation during flow of air-water mixtures in smooth and rough horizontal tubes. Improvements in the two-phase flow correlations for rough tubes are presented. Approximate empirical relationships developed using these improvements correlated the majority of the data within 15%.

From authors' summary

2685. McManus, H. N., Jr., An experimental investigation of film characteristics in horizontal annular two-phase flow, ASME Ann. Meet., New York, N. Y., Dec. 1957. Pap. 57-A-144, 17 pp.

Annular two-phase flow in a horizontal tube was established by injecting water through an annular slot in the tube wall into a fully developed turbulent air flow. The film-profile dimensions, the liquid-disturbance height, the two-phase pressure drop, and the extent of film establishment were measured. The air Reynolds number was varied from 75,000 to 125,000. Water flows up to 0.7 lb/sec were used. The effect of flow rates upon liquid surface-disturbance character was examined visually and photographically. The profile dimensions throughout the tubes were found to be sensitive to air rate. The liquid rate varied the film depth only in the lower half of the tube. Disturbance height was found sensitive to the air rate and the liquid depth. The pressure drop before the establishment of steady-state conditions was sensitive to the inlet-film thickness. Film establishment exhibited a similar dependence. Photographic examination of the film-surface characteristics showed a dependence upon water rate as well as air rate. A correlation was obtained for the film thickness in the upper two thirds of the tube.

From author's summary

2686. Taylor, Sir G., Fluid dynamics in a papermaking machine, *Proc. roy. Soc. Lond.* **242,** 1228, 1-15, Oct. 1957.

The analysis of incompressible flow through a porous cone-shaped wall [AMR **9** (1956), Rev. 4127] is here extended to the wedge-like space formed by a band of wire gauze in horizontal motion (U) in contact with a cylindrical roller. Such a band supported by several rollers transports the pulp in paper-making machinery and drains off most of its water, mainly by hydrodynamic suction.

Under suitable simplifying assumptions, mass conservation and Bernoulli's equation lead, for laminar flow, to an integral equation for the underpressure as a function of the distance from the contact point (in the direction of the motion). Numerical solutions are presented for linear and quadratic resistance characteristics of the gauze; in both cases the peak underpressure is $\rho U^2/2$. For complete turbulent mixing (= no vertical velocity gradient), momentum equation and mass conservation lead to a peculiar boundary value problem for a nonlinear resistance law; here the peak underpressure is $1.4\ \rho U^2/2$.

These results are compared with earlier calculations by Wrist and with some experiments that more nearly fit Wrist's assump-

tions but also with experiments approximating the conditions in paper making. G. Kuerti, USA

2687. Berman, A. S., Laminar flow in an annulus with porous walls, *J. appl. Phys.* **29,** 1, 71–75, Jan. 1958.

Solution of Navier-Stokes equations is obtained to provide velocity and pressure variations for flow in an annulus with porous walls. For special case of constant influx through one wall equal to efflux through other, results for combined longitudinal and transverse flow are expressed in terms of ratio of radii σ, longitudinal pressure gradient, and Reynolds number R of transverse flow. Inflection point occurs in velocity distribution if flow is outward and if R is above critical value which depends on σ. Author suggests effect of inflection on stability of flow.

J. S. McNown, USA

2688. Burgers, J. M., and Ghaffari, A., On the application of steam-driven water jets for propulsion purposes, *J. Res. nat. Bur. Stands.* **60,** 2, 137–141, Feb. 1958.

Calculations have been made concerning the momentum that can be given to a jet of water by mixing it with a jet of high-speed steam. Apart from the application of the equation of momentum and enthalpy this raises questions concerning the speed of condensation and the acceleration of the water.

From authors' summary by S. I. Pai, Germany

2689. English, R. D., Spinak, A., and Helton, E. H., Physical characteristics and test conditions of an ethylene-heated high-temperature jet, *NACA TN* 4182, 28 pp., Jan. 1958.

A ground test facility has been developed at the Langley Pilotless Aircraft Research Station at Wallops Island, Va., that is capable of producing temperatures up to nearly 5,000 R. Conditions in the test section obtained from the calibration runs and from the calculated thermal properties of the exhaust gas are presented over the range of stagnation temperatures available.

From authors' summary

2690. Consiglio, J. A., and Sliepcevich, C. M., Effect of liquid physical properties and flow rates on the surface area of sprays from a pressure atomizer, *AIChE J.* **3,** 3, 418–427, Sept. 1957.

The title problem was investigated experimentally; the specific surface area of the sprays is correlated by an equation of two dimensionless groups in terms of the variables: (a) surface tension to the -1.0 power, (b) kinematic viscosity to the -0.4 power, and (c) volume flow rate to the 2.4 power. The volume flow rate is correlated by an equation of two dimensionless groups containing the variables: (a) viscosity to the 0.17 power, (b) density to the -0.58 power, and (c) spray pressure to the 0.42 power. The conversion of compression energy to surface-area energy appears to be constant at approximately 0.1%. An optical sampling method is used based on the light-scattering properties of spherical particles. The method consists of measuring continuously the intensity of a transmitted light beam through a dispersion of droplets as they settle differentially under the influence of gravity through a known settling distance. The size distribution of the spray is calculated from the light-intensity measurements by means of a modified form of the Lambert-Beer transmission equation, and the Stokes Law. The technique is restricted to drops of less than 40-micron diam. Method of calculation, tables of numerical data, and their graphical representation are explained and given in detail. Attempt was made to check the surface-area values by means of high-speed photographic technique but it was not successful owing to the high velocity of the droplets issuing from the nozzle.

K. J. DeJuhasz, Germany

2691. Roy, A. K., Study of the curving of the free jet, *J. aero. Soc. India* **9,** 2, 28–33, May 1957.

The problem of curving of the free jet has been studied by determining the form of the free streamline, breadth of the jet, curvature of the spiral into which the free streamline winds up, etc., by the help of momentum theorem, continuity and energy equations, conformal transformation, etc. The idea of curving of the free jet has been derived from the diffusor.

From author's summary

2692. Buchmann, S. W., An experimental investigation of drop disintegration (in Russian), *Vestnik Akad. Nauk KazSSR.* no. 11, 80–87, 1955; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7449.

The experimental method and results are presented of an investigation of the process of disintegration of liquid drops in an airstream.

The drops are photographed by stroboscopic illumination, enabling both the velocity and the instant of disintegration to be determined.

The experiments were conducted with drops of approximate spherical shape measuring $1.39 \leqslant d \leqslant 2.98$, run in a range of flow velocities between $8.4 \leqslant \omega \leqslant 11.35$ mps.

In the author's opinion, calculation of the fragmentation factor of the drops requires taking into account the velocity of the particle at the instant of disintegration.

The numerical value of the factor is found to be influenced by turbulence in the flow.

The values of the fragmentation factor obtained in the experiments diverge considerably from the values previously determined by other authors. [Prandtl: "Hydro-Aerodynamics," Foreign Literature Publishing House, 1951; M. S. Volynsky, *Dokladî Akad. Nauk SSSR* **68,** 2, 237–240, 1949; A. S. Zverev, B. V. Kiryukhin, and others, "Textbook of meteorology," Tver Hydro-Meteorological Publications, 1951.] M. S. Volynskii

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2693. Bisa, K., Birnagl, K., and Esche, R., Atomization of liquids by ultrasonics (in German), *Siemens-Zeit.* **28,** 8, 341–347, 12 figs. + 12 ref., Sept. 1954.

An ultrasonic aerosol generator for inhalation therapy has been investigated for the influence of the characteristics of the ultrasonic field (frequency, power) on the characteristics of the aerosols produced (droplet size distribution and mean droplet size, and quantity of fog produced), using various liquids. The aerosol generator consisted of a barium-titanate oscillator of concave spherical shape having an included cone angle of about 60 deg, acting upon the liquid and producing a focused ultrasonic field therein. Thereby a high-intensity field is produced at relatively low power. The influence of liquid height on the rate of atomization (cc/min) was investigated for water and for gasoline. The droplet size and fog density were determined for a brine of 5% salt content at frequencies of 1.2, 1.7, 2.7, and 5.4 megacycles at various powers up to 0.6 kilowat; experimental arrangement and curves of results are shown. A chart is given for droplet size distribution of aerosols produced by various methods (by nozzles and by ultrasonics). It was found that the mean droplet size was reduced significantly by increasing the frequency, and reduced (slightly) by increasing the power. The amount of liquid atomize (cc/min) was greater in the case of gasoline (low surface tension) than in the case of water (high surface tension). The experimental results were evaluated as to the influence of cavitation, radiation pressure, surface phenomena, frequency and power of ultrasonic energy input, and efficiency of atomization. The effect of ultrasonic irradiation on macromolecular substances, in particular on antibiotics (aureomycin, digitalis) from the point of their biological effectiveness, is discussed. K. J. De Juhasz, Germany

Compressible Flow, Gas Dynamics

(See also Revs. 2474, 2664, 2724, 2731, 2732, 2733, 2735, 2741, 2764, 2765, 2820, 2835)

2694. Fasoli, U., Theory of vacuum formation with steam-jet ejectors (in Italian), *Termotecnica* **11**, 9, 465–472, Sept. 1957.

After a brief summary of the variables entering the problem and elementary theory of ejectors, author applies equation of motion of compressible fluid and concludes that discharge is a function of both aspiration and downstream pressures. Discharge is also a function of steam pressure. There is no limit for increase of discharge with increase in steam pressure: the higher the steam pressure is, the less specific steam consumption is required and the degree of vacuum obtained is better. Both fluids have independent energy balances and are only partially blended. Author gives some procedures for computation of an ejector and concludes with a comparison between the performances of pumps and ejectors.

A. Balloffet, USA

2695. Kuene, F., and Schmidt, W., Invariance criteria for subsonic and supersonic flow on equivalent bodies with special regard to wing body combinations (in German), *Jahrbuch Wissenschaft. Gesellsch. Luftfahrt,* 1956, 150–155.

It is shown, according to slender and not so slender wing and body theory at transonic speeds, that differences in the velocity on equivalent bodies and wing-body combinations at the same incidence at two given Mach numbers are a function of body shape only.

For notation refer to earlier work; e.g. AMR **7** (1954), Rev. 2527.

G. M. Lilley, England

2696. Koga, T., A method for solving problems of irrotational gas flow by means of high-speed digital computers, *J. appl. Mech.* **24,** 4, 497–500, Dec. 1957.

Method is described for integrating compressible flow equations numerically by proceeding from one streamline to the next. Advantage of method is that it is applicable to mixed subsonic and supersonic flow. Disadvantage is that both a streamline and the state of the gas along that streamline must be given as boundary conditions. Successive approximations must be used to extend the method to other types of boundary conditions. Treatment is limited to two-dimensional shockless potential flow.

J. M. Hedgepeth, USA

2697. Page, W. A., Experimental determination of the range of applicability of the transonic area rule for wings of triangular plan form, *NACA TN* 3872, 22 pp., Dec. 1956.

Drag measurements were made in the Ames 2 × 2-ft transonic tunnel in the region $M = 0.6$ to 1.4 on six wing-body models under conditions of zero lift with Reynolds number 1 million. Triangular wings with A between 2.0 and 5.6 and aerofoil sections NACA 63 A00x were mounted on a slender cylindrical body and strain-gage balance technique employed. Author is clear and precise in discussing the conditions and assumptions necessary to the reduction of the data.

Author's results show that the data are in agreement with the transonic area rule for values of $A(t/c)^{1/3}$ up to 1.3 (agreement is not bad up to about 2) compared with values up to 1.0 for rectangular wings. An appendix discusses body-nose tunnel interference under conditions of test.

R. C. Knight, England

2698. Guderley, G., On the development of solutions of Tricomi's differential equation in the vicinity of the origin, *J. rational Mech. Analysis* **5,** 5, 747–790, Sept. 1956.

Paper gives a purely mathematical treatment of Tricomi's equation $\psi_{\eta\eta} - \eta\psi_{\theta\theta} = 0$ satisfied by the stream function ψ of plane transonic potential flow as a function of η, which is proportional to the deviation of the absolute value of the velocity from the sonic velocity, and of θ which is the angle of the velocity vector with a fixed axis. There exists a family of solutions, called the first family, of the form $\psi = |\eta|^n f(\zeta)$ where $\zeta = 9\theta^2/(4\eta^3)$, fulfilling the boundary conditions that ψ and its derivatives be finite along the characteristics $\theta = \pm(2/3)\eta^{3/2}$. Detailed formulas for these particular solutions, including their asymptotic representations for large values of n are given. Nothing is known about the completeness of this system. A second family of particular solutions is introduced, defined by an eigenvalue problem with modified boundary conditions. For this family the completeness follows from Hilbert's theory of the polar integral equation. A formula is derived representing an arbitrary function of one variable in terms of the eigenfunctions of this second family. With its aid it is possible to solve a certain inhomogeneous boundary-value problem which requires the determination of a solution ψ of Tricomi's equation for which the jump of ψ and of a suitable outer derivative is prescribed on a given curve of the η, θ-plane. The solution of this inhomogeneous problem appears first in a rather complex form using the second family of particular solutions, but by means of the theory of functions of a complex variable and by deforming the path of integration in some integrals occurring there, it finally is transformed in such a manner that it appears as a superposition of the particular solutions of the first family. It thus gives the arbitrary solution of Tricomi's equation under certain restrictive conditions which concern the region of convergence of this representation.

H. Behrbohm, Sweden

2699. Legendre, R., Law of flow of supersonic area rule (in French), *Rech. aéro.* no. 50, 3–8, 1956.

The wave drag of the supersonic flow around an airfoil or a slender body indicates quasi-incompressibility of the flow along the Mach lines. This suggests that the wave drag depends primarily upon the law of variation of cross-sectional area of the body as a function of distance along the body. An improvement of the rule, useful for the study of slender bodies with wings flying at supersonic speeds, is obtained by substituting for the cross-sectional areas the areas included in the projection on a transverse plane of the intersection of the body with the Mach cone.

The extended supersonic area rule does not relieve one from calculating the flow around the body. However, it gives an order of magnitude of the wave drag and hints at ways to reduce the wave drag.

From author's summary by G. D. Boehler, USA

2700. Tan, H. S., On optimum nose curves for missiles in the superaerodynamic regime, *J. aero. Sci.* **25,** 1, 56–57 (Readers' Forum), Jan. 1958.

Author shows that the differential equations defining the minimum drag body shapes for free molecule flow that were developed and numerically integrated by W. J. Carter [AMR **11** (1958), Rev. 207] can be integrated analytically. Neither the author nor Carter realized, however, that numerical or analytical integration of the second-order differential equation is unnecessary since, for the flow conditions considered, the first integral to the Euler equation can be written prior to the substitution of the expression defining the pressure coefficient.

D. H. Dennis, USA

2701. Chang, I. D., On optimum nose shapes for missiles in the superaerodynamic region, *J. aero. Sci.* **25,** 1, 57–58, Jan. 1958.

Author shows that the differential equation defining one of the minimum drag body shapes for free molecule flow that were developed and numerically integrated by W. J. Carter [AMR **11** (1958), Rev. 207] can be integrated analytically, and he solves exactly for the body shape. Neither the author nor Carter realized, however, that integration of the second-order differential equation is

unnecessary since, for the flow conditions considered, the first integral to the Euler equation can be written immediately and the body coordinates in terms of body slopes readily deduced.

D. H. Dennis, USA

2702. Serbin, H., Supersonic flow around blunt bodies, *J. aero. Sci.* **25,** 1, 58-59 (Readers' Forum), Jan. 1958.

Approximate analysis has been made of shape of shock wave ahead of sphere and flat-nosed body of revolution. Results are summarized and reinterpreted in terms of density ratio across normal shock. Reviewer notes that although solution is based on well-known Busemann hypersonic limit, resulting standoff distance for sphere is only 2/3 of that established by Hayes ["Some aspects of hypersonic flow," Ramo-Wooldridge Corp., Jan. 1955], Chester [AMR **10** (1957), Rev. 3696], Freeman [AMR **10** (1957, Rev. 3704], and Li and Geiger [AMR **10** (1957), Rev. 2979]. Hence good agreement shown with experiment may be fortuitous.

M. D. Van Dyke, USA

2703. Gadd, G. E., A theoretical investigation of laminar separation in supersonic flow, *J. aero. Sci.* **24,** 10, 759-771, 784, Oct. 1957.

Effects of Mach number, Reynolds number, wall temperature, and surface curvature on laminar separation in supersonic flow are investigated by an extension of Stratford's method for incompressible flow. It is assumed that the flow is two-dimensional, and that where the surface is curved, the generators are perpendicular to the flow, as with a two-dimensional airfoil. The wall temperature is taken to be uniform, though in principle the method can still be applied if the wall temperature varies from point to point, provided that its gradient in the region of separation is not too great. It is assumed that separation is of the usual practical type for supersonic flow, taking place well upstream of shock wave or other agency which provokes separation. In these circumstances, the flow in the neighborhood of separation is entirely governed by Mach number, Reynolds number, wall temperature, and surface curvature. The results of the analysis for characteristics of the flow such as the pressure distribution at the wall agree well in most respects with experiment.

From author's summary by S. Tomotika, Japan

2704. Coburn, N., Intrinsic form of the characteristic relations in the steady supersonic flow of a compressible fluid, *Quart. appl. Math.* **15,** 3, 237-248, Oct. 1957.

Characteristic relations are expressed in intrinsic form for steady, supersonic, three-dimensional, rotational and irrotational motion of a polytropic gas. These relations are applied to the study of a class of Beltrami flows characterized by one family of characteristic surfaces consisting of a single infinity of parallel planes.

D. R. Chapman, USA

2705. Munch, J., Contribution to the source-sink method for the calculation of supersonic flow (in German), *ZAMM* **37,** 1/2, 51-63, Jan./Feb. 1957.

Paper deals with supersonic flow around pointed slender bodies of revolution. Usual linear approximations are used and solutions are found with the help of the source-sink method. Different approaches are discussed, and the method is generalized for the case when the contour has noncontinuous derivatives. One numerical example is given.

T. R. Gullstrand, Sweden

2706. Keune, F., Series expansion of velocity potentials in linear sub- and supersonic flow for not-so-slender bodies (in German), *Z. Flugwiss.* **5,** 8, 243-247, 1957.

Velocity potentials for sub- and supersonic flows over symmetrical not-so-slender bodies are obtained from a simple mathematical derivation. Formulas for each of the terms for $M < 1$ and $M > 1$ are analogous to the terms corresponding to pure two-dimensional sources and "space influences." The error at a given stage of approximation is proportional to the first term in the series expansion not used.

W. C. Griffith, USA

2707. Morioka, S., Note on the flow just behind three-dimensional shock, *J. Japan Soc. aero. Engng.* **4,** 35, 311-316, Dec. 1956.

Expressions are given for the derivatives of flow quantities behind a steady three-dimensional shock. The expressions are comparatively simple if the parameters are the lengths of the radii of curvature.

From author's summary by S. H. Maslen, USA

2708. Jukes, J. D., The structure of a shock wave in a fully ionized gas, *J. Fluid Mech.* **3,** 3, 275-285, Dec. 1957.

2709. Meyer, R. F., The impact of a shock wave on a movable wall, *J. Fluid Mech.* **3,** 3, 309-323, Dec. 1957.

An approximate solution is devised for the one-dimensional motion following the impact of a shock wave on a wall which is free to move. The asymptotic behavior of the system is considered and it is shown by exact physical argument that the transmitted shock eventually attains the same strength as the incident shock and that the reflected shock ultimately decays to a sound wave.

An experimental investigation of the interaction was made, using thin walls of cellulose acetate, in a shock tube at an incident shock Mach number of 1.50. Agreement between the theoretical and experimental results, especially for the path followed by the wall, was found to be good.

From author's summary by J. V. Becker, USA

2710. Page, R. H., Sidewall influence on oblique-shock pressure, *J. aero. Sci.* **24,** 10, 777-778, Oct. 1957.

2711. Garabedian, P. R., Numerical construction of detached shock waves, *J. Math. Phys.* **36,** 3, 192-205, Oct. 1957.

Author considers the numerical solution of the problem of a detached shock wave upstream of an obstacle in supersonic flow. As previously considered, this Cauchy problem encounters a familiar difficulty with accuracy since the difference equation analog is unstable. Author reformulates the problem by analytic continuation of the space variables x and y into the domain of complex variables. In this form the problem leads to hyperbolic differential equations which have stable difference analogs. Author solves the inverse problem of determining the body shape when the shock profile is given as an analytic function. In particular, he gives numerical results for a shock front of parabolic form.

Author points out that the success of his method requires that the initial data on the shock curve must not possess any singularities near or in the real domain (See following review).

M. S. Plesset, USA

2712. Lin, C. C., Note on Garabedians's paper "Numerical construction of detached shock waves," *J. Math. Phys.* **36,** 3, 206-209, Oct. 1957.

Author applies Garabedian's procedure of analytic continuation to the elliptic initial-value problem of the type considered by Hadamard. In the example presented by the author, a solution is given of the Laplace equation in which the dependence of the solution and the initial conditions on an arbitrary parameter is given explicitly. It is shown that for a limiting value of this parameter the limit of the solution no longer matches the limiting values of the initial conditions. Author then shows for this case that this instability persists with the analytic continuation procedure of Garabedian.

It would appear, however, that Garabedian is aware of limitations in his procedure in that it cannot be applied to singular conditions. Present author does point out the need for more extensive examination of the conditions under which accurate solutions may be found by Garabedian's procedure (See preceding review). M. S. Plesset, USA

2713. Daskin, W., and Feldman, L., The characteristics of two-dimensional sails in hypersonic flow, *J. aero. Sci.* **25,** 1, 53–55, Jan. 1958.

The aerodynamic and geometric characteristics of two-dimensional sails in a hypersonic flow are investigated. These surfaces would have very large lift and drag per unit weight. Therefore they appear attractive whenever very large values of the parameters $C_L A/W$ or $C_D A/W$ are needed. Simple closed expressions are derived for the sail shape and the corresponding lift, drag, and moment coefficients. Charts of these quantities are presented for the entire useful design range. For the same materials, stresses, and planforms, sails are claimed to have 1/5 to 1/10 the weight of conventional wing.

From authors' summary by H. P. Liepman, USA

2714. Zlotnick, M., and Newman, D. J., Theoretical calculation of the flow on blunt-nosed axisymmetric bodies in a hypersonic stream, Avco, Research and Advanced Development Div., TR-2-57-29, Sept. 1957.

Title problem has been solved by means of a rapid approximate calculation in the case of a flat-nosed cylinder. The inverse problem of determining the body from the given shock has been solved by a numerical stepwise integration method. The first calculation for the shock detachment distance has been compared with experimental results. The results of the second method have not been compared with experimental results.

Reviewer believes that there is satisfactory agreement between the approximate calculation and the numerical method, but the agreement with experiments requires further investigation, particularly for $k > 0.25$. Y. V. G. Acharya, India

2715. Bertram, M. H., and Barsdell, D. L., A note on the sonic-wedge leading-edge approximation in hypersonic flow, *J. aero. Sci.* **24,** 8, 627–629 (Readers' Forum), Aug. 1957.

2716. Sedney, R., Shock wave curvature for hypersonic axisymmetric flow, *J. aero. Sci.* **24,** 8, p. 630 (Readers' Forum), Aug. 1957.

2717. Oliver, R. E., and Cummings, B. E., The effect of a simple throat distortion on the flow in a hypersonic wind-tunnel nozzle, *J. aero. Sci.* **24,** 6, 466–467 (Readers' Forum), June 1957.

Wave Motion in Fluids

(See also Revs. 2477, 2656, 2707, 2765, 2884, 2900, 2901, 2902)

2718. Kaplan, P., The waves generated by the forward motion of oscillatory pressure distributions, Proc. Fifth Midwestern Conf. Fluid Mech., Univ. of Mich., Apr. 1957, 316–329.

This paper considers the wave system generated by a moving distribution of pressure on a water surface initially at rest. Author uses a Fourier integral approach and a linearized two-dimensional irrotational fluid of infinite depth.

The pressure source is a localized pressure in space and with a frequency $\omega/2\pi$; it moves along the x axis with velocity V.

It is shown that for large values of $|x|$ there will be wave trains extending to great distance ahead of the source provided that $\omega V/g$ is less than 1/4, and that no wave train will be found ahead of the source if $\omega V/g$ is greater than 1/4. Author gives calculations in the case of a pressure point source (delta function) and a vertical dipole source. The method can be extended to more extended sources and to three-dimensional problems. For the technique of dealing with the contour integrals involved cf. E. R. Lapwood, *Phil. Trans. roy. Soc. Lond.* (*A*) **242,** p. 63, 1949: "Disturbance due to a line source in a semi-infinite elastic medium." J. M. Jackson, Scotland

2719. Suter, P., Pressure wave with laminar friction (in German), *Motortech. Z.* **18,** 8, 247–251, Aug. 1957.

The laws of propagation of pressure and velocity waves in circular pipes of constant cross section throughout have been studied for the injection system of diesel engines. Assuming a power law for the axial velocity $c = C_{max}\,[1 - (r/R)^n]$, where r is the distance from the axis and R is the radius of the pipe, the equations of momentum and continuity have been set up for the axial variation of quantities according to linearized compressible flow theory. The results are compared with the friction-free case, and graphs of pressure and velocity versus time are given.

S. D. Nigam, India

2720. Loginov, V. N., The deformation of sea waves in shallow water (in Russian), *Trudī Centr. n.-i. in-ta mor. flota* **1,** 3, 54–70, 1955; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8301.

The causes of wave deformation in shallow water are examined and summary recommendations made for the design of coast defence works, as well as the prediction of the movement of deposits in the littoral zone.

It is pointed out that the fundamental difficulty consists in phenomena of refraction, which are practically impossible to analyze mathematically. An exception is presented by a few cases of an academic nature, investigated by Yu. M. Krylov, which, as the author points out, have no great practical significance.

Author presents a number of approximate graphical methods for calculating the change in the angle of refraction and variation of the wave elements, in shallow water.

It is further pointed out that the causes of loss of energy in the waves are associated with the influence of the boundary layer and the presence of filtrational currents. For the evaluation of the quantity characterizing the dissipation of energy, author proposes the following expression

$$P_T = \frac{\gamma K_s}{T}\int_0^T \xi^3 dt$$

in which ξ is the velocity of the fluid particles at the bottom, calculated from the linear theory of travelling waves over a smooth surface; γ is the specific gravity of the water, K_s the coefficient of roughness of von Mises, T the period of oscillation.

The rate of dissipation of energy by filtration is suggested to be calculated according to Kondratyev's formula

$$P_\Phi = \frac{\gamma\pi K_\Phi h^2}{4\lambda \mathrm{ch}^2 KH}\, \mathrm{th}KL$$

in which L = depth of penetrated layer of the bottom, K_Φ d'Arcy coefficient, h wave height, λ wave length, H depth of water.

The last part of the paper describes methods of determining the depth at which the waves begin to decay.

N. N. Moiseev

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2721. Fabula, A. G., Ellipse-fitting approximation of two-dimensional, normal symmetric impact of rigid bodies on water, Proc. Fifth Midwestern Conf. Fluid Mech., Univ. of Mich., Apr. 1957, 299–315.

The problem of normal symmetric wedge impact is considered. The ellipsoid fitting approximation used by Shiffman and Spencer, *Comm. pure appl. Math.* **4**, p. 379, 1951, was adapted to this problem.

When results were compared with Wagner's formula, *ZAMM* **12**, 1932, the ellipse-fitting method gave better results than a more accurate diamond-shaped wedge. Author suggests that more accurate predictions of impact loads can be made by further extensions of the ellipse-fitting methods.

J. M. Jackson, Scotland

Turbulence, Boundary Layer, etc.

(See also Revs. 2665, 2679, 2680, 2775, 2781, 2787, 2822, 2884)

2722. Mickley, H. S., and Davis, R. S., Momentum transfer for flow over a flat plate with blowing, *NACA TN* 4017, 64 pp., Nov. 1957.

Paper reports on one phase in an extended study on momentum and heat transfer through laminar and turbulent boundary layers with blowing of a coolant gas through the surface. Velocity profiles, boundary-layer thicknesses, and friction factors are reported for a turbulent air boundary layer on a flat plate with injection of air through the porous surface. The results are compared with a theory by M. W. Rubesin [*NACA TN* 3341, 1954; AMR **8** (1955), Rev. 2139] based on the mixing length concept. Good agreement was obtained when the thickness of the laminar sublayer was permitted to vary with the injection velocity. The experimental results partially supersede previously reported measurements in which inconsistencies had been detected.

E. R. G. Eckert, USA

2723. Weske, J. R., and Chen, Y. Y., Calculation of the cross-stream wave length of the three-dimensional transition pattern and correlation with the Gortler instability theory, *Inst. Fluid Dynam. appl. Math., Univ. Maryland, AFOSR-TN* 57-637, 22 pp. + 8 figs., Oct. 1957.

A theory is presented which deals with the mechanism producing a regular three-dimensional pattern at an early stage of transition in the boundary layer. Relations for secondary vorticity, for the case of rotational yet inviscid flow, are used to derive quantitative expressions for the cross-stream wave length of this pattern for uniform and for retarded free flow. The results obtained are linked to the Taylor-Görtler theory of instability along curved walls.

From authors' summary by S. Ostrach, USA

2724. Lessen, M., On the hydrodynamic stability of curved laminar compressible flows, Proc. Fifth Midwestern Conf. Fluid Mech., Univ. of Michigan, Apr. 1957, 22–28.

The stability of curved laminar compressible flows is examined and it is found that instabilities in the flow fields about bodies moving at high speed may exist under certain circumstances.

The general disturbance equations are derived, and simplified cases are discussed. Whereas in the case of incompressible flows the instability is due to the kinematics of the flow field, for the compressible case, thermodynamic factors are also of importance.

From author's summary by D. R. Chapman, USA

2725. Esch, R. E., The instability of a shear layer between two parallel streams, *J. Fluid Mech.* **3**, 3, 289–303, Dec. 1957.

Paper contains a theoretical investigation into the growth of small periodical disturbances in a shear layer between two parallel streams. If the difference of the velocities of the streams is $2U$, the width of the boundary layer $2H$, and the kinematical viscosity is ν, the corresponding Reynolds number is $R = UH/\nu$. The region of instability in the plane of Reynolds number R and disturbance wave number α is determined, and typical growth rates in the unstable region are computed. Both analytical and numerical methods are used. The asymptotical expansions are found to give good results for $\alpha RH > 100$ only. An analytical method useful for $\alpha RH < 1$ is developed. Unstable disturbances are found at all values of R. Results for $\alpha RH > 100$ are found to agree closely with inviscid theory results.

These results are perhaps not quite unexpected, but hydrodynamics is so full of surprises that such an excellently performed calculation as this is welcome.

H. Faxen, Sweden

2726. Lighthill, M. J., The fundamental solution for small steady three-dimensional disturbances to a two-dimensional parallel shear flow, *J. Fluid Mech.* **3**, 2, 113–144, Nov. 1957.

Paper gives solution for a weak point source in a two-dimensional parallel, not necessarily uniform, shear flow of the form $(V(y), 0, 0)$; that due to a doublet may be obtained by differentiation; more complicated solutions can be built up from these. Viscosity is neglected, which is permissible if it is supposed that $V(y)$ nowhere vanishes.

Since the source is weak, squares and products of disturbances from the above flow field are neglected in the Oseen manner. Near the source the first approximation to the velocity vector is of order r^{-2}; the second, of order r^{-1}, depends on $V(0)$ and $V'(0)$; in fact, for order r^{-1} the solution is the same as though the shear were uniform.

At large distances the velocity field is given in terms of order r^{-2} and r^{-3}, with an error term of order r^{-4}. It is also shown that behavior for large positive y is the same as that due to a source of different strength in a different position. The solutions for small and large distances overlap and so the whole field is covered.

The solution is also given for the case when the shear is such that the total variation of velocity is small.

Application is made to the displacement of the stagnation streamline of a pitot tube in shear flow, in particular to the downwash at points far ahead of the tube.

Mathematics is elegant and concise, yet simple and clear. Exposition is not as economical in words as has been fashionable recently, but it gains greatly by this. Tendencies since the World War II have been toward more and more brevity, leading to more and more unintelligibility. It is time to reverse this trend.

J. C. Cooke, England

2727. Tichacek, L. J., Barkelew, C. H., and Baron, T., Axial mixing in pipes, *AIChE J.* **3**, 4, 439–442, Dec. 1957.

Authors derive improved values of the axial mixing coefficient for incompressible, turbulent flow in a circular cylinder. The new values are estimated from G. I. Taylor's analysis refined to include the effects of molecular diffusion and also to include experimental rather than calculated velocity profiles. Over the Reynolds number range from 2.6×10^{3} to 2.8×10^{6}, these new computed mixing coefficients, which exhibit great sensitivity to the shapes of the velocity profiles used in the calculations, are in better agreement with experimental results than the mixing coefficients computed by Taylor. Authors' results enable improved estimates of the dispersion of tracers and of the intermixing of products in a pipeline, but these results are subject to errors less than 25% only for characteristic concentration lengths greater than 50 to 100 pipe diameters.

H. A. Stine, USA

2728. Schubauer, G. B., and Klebanoff, P. S., Contributions on the mechanics of boundary-layer transition, *NACA Rep.* 1289, 11 pp., 1956.
[See AMR **9**, Rev. 507]

2729. Moore, F. R., Lift hysteresis at stall as an unsteady boundary-layer phenomenon, *NACA Rep.* 1291, 10 pp., 1956.
[See AMR **9**, Rev. 3341]

2730. Cohen, C. B., and Reshotko, E., Similar solutions for the compressible laminar boundary layer with heat transfer and pressure gradient, *NACA Rep.* 1293, 38 pp., 1956.
[See AMR **8**, Rev. 2807]

2731. Moeckel, W. E., Some effects of bluntness on boundary-layer transition and heat transfer at supersonic speeds, *NACA Rep.* 1312, 14 pp., 1957.
[See AMR **9**, Rev. 1892]

2732. Bertram, M. H., Exploratory investigation of boundary-layer transition on a hollow cylinder at a Mach number of 6.9, *NACA Rep.* 1313, 27 pp., 1957.
[See AMR **9**, Rev. 2998]

Aerodynamics of Flight; Wind Forces

(See also Revs. 2572, 2573, 2666, 2667, 2678, 2713, 2729, 2767, 2768, 2769, 2770, 2834, 2836)

2733. Helliwell, J. B., and Mackie, A. G., Two-dimensional subsonic and sonic flow past thin bodies, *J. Fluid Mech.* **3**, 1, 93–109, Oct. 1957.

The flow past symmetric airfoils with sharp shoulders is studied with the aid of the simplifying assumption that the flow separates from the surface at the shoulder instead of performing a Prandtl-Meyer expansion around the corner. Hodograph methods are employed and the usual approximations of transonic flow theory are introduced to reduce the fundamental equation for the stream function to Tricomi's equation. A solution is obtained in terms of Bessel functions. Specific results are given for a wedge profile and for sonic flow past a profile that can be represented by a first-order perturbation upon a wedge. It is shown that the results for sonic flow past a wedge are very similar to the corresponding existing results for unseparated flow past the same profile.

The present paper is a valuable contribution to transonic flow theory since it presents an exact solution for a representative problem of thin airfoil theory against which approximate solutions can be tested. Reviewer calls attention to the fact that a very simple approximate solution that leads to very similar numerical results for the same problem has been published recently by Kusukawa in the *J. phys. Soc. Japan* **12**, 9, 1031–1041, Sept. 1957. The latter results are determined by application of a pressure-correction formula that relates the pressures in a compressible flow to those in an incompressible flow. The same formula, except for replacement of γH by $M_\infty^2(\gamma + 1)$ as a result of a corresponding change in the fundamental differential equation of transonic flow theory, has also been found independently by the reviewer by application of much different considerations and presented in *NACA TN* 3970. Incorporation of such a change into the results given in both of the mentioned papers would yield a substantial improvement in the accuracy of the results for Mach numbers other than one, particularly at the shoulder and along the separated streamline.

J. R. Spreiter, USA

2734. Eckhaus, W., A method for the asymptotic expansion of the integral equation of a lifting surface at near-sonic speeds, *Nat. LuchtLab. Amsterdam Rap.* F.200, 15 pp., Apr. 1957.

An asymptotic expansion of the linearized integral equation for steady subsonic flow is obtained. Provided the Mach number is near unity, this expansion is valid for general wing planforms of low aspect ratio except near kinks in the leading or trailing edges. The procedure is illustrated by considering a delta wing planform, where the solution obtained is reasonably accurate except in the vicinity of the trailing edge.

A. R. Mitchell, Scotland

2735. Keune, F., Systematic considerations on sub- and supersonic flow around not-so-slender bodies (in German), *Z. Flugwiss.* **5**, 4, 121–125, Apr. 1957.

The expression for the velocity potential for the flow around wings of small and moderate aspect ratio is expanded in a series and the significance of the terms usually disregarded in slender wing theory is discussed in terms of moments of area of the cross-section shape. A number of conclusions of theoretical and practical interest are stated briefly, but publication of the details is deferred to a subsequent paper.

J. R. Spreiter, USA

2736. Lattanzi, B., Diagrams for calculating the incremental velocities for sweptback wings with double curvature symmetrical profiles and subsonic leading and trailing edges (in Italian), *Aerotecnica* **37**, 1, 7–12, Feb. 1957.

Paper is concerned with the computation of the local pressure coefficients on triangular wings having subsonic leading and trailing edges and symmetrical curvilinear profiles. The method is based on the conventional linearized equations using supersonic sources of intensity proportional to the local slope of the profile. A certain number of integrals has been derived, which have been calculated for various values of the maximum thickness line position and for various values of the parameter $B = (M^2 - 1)^{1/2}$ (where M is the Mach number), and for three leading-edge sweep angles (30°, 45°, 60°). The relative plottings are presented.

It is shown how, by using such plottings, it is possible to calculate velocities and pressure coefficients in all the points of the wing. The method is valid only for thickness drag computations, since the zero angle of attack only has been considered.

C. Buongiorno, Italy

2737. Barna, P. S., Aerodynamic characteristics of a 40° sweptback wing of aspect ratio 4.5, *Coll. Aero. Cranfield Rep.* no. 65, 5 pp. + 8 figs., May 1957.

Experimental investigations have been made of the flow characteristics on wings of moderate sweepback. This note presents the results of the experimental investigation into the pressure distribution over a sweptback wing having an aspect ratio 4.5 and sweepback angle 40° at quarter chord. A half-model technique has been used. The wing section was R. A. E. 101 (symmetrical) with 6% maximum thickness and zero camber. Measurements of chordwise pressure distributions at a number of spanwise sections were made in the incidence range 2–32°. The experiments were performed at Reynolds numbers of 0.9×10^6 and 2×10^6 (based on geometric mean chord). From these measurements, lift, drag and pitching moment characteristics of the wing were calculated. Results of the experiments for small incidences have been compared with the calculated loadings obtained by methods proposed by Kucheman. The theory predicts higher values in the spanwise load distribution at the root stations and a lift-curve-slope 8–12% lower than obtained in the experiments.

From author's summary

2738. Van Spiegel, E., Theory of the circular wing in steady incompressible flow, *Nat. LuchtLab. Amsterdam Rap.* F. 189, 52 pp. + 4 tables + 4 figs., Jan. 1957.

The problem of the circular airfoil previously studied by Kinner, Schade, Krienes and Kochin, is reconsidered by the author as a

boundary-value problem for the Laplace equation. The complete solution is given as the sum of the regular acceleration potential together with a term which is singular only along the leading edge of the wing and which contains an unknown weight function. The resulting integral equation is replaced by an infinite system of linear algebraic equations which are broken off in the usual way. The results are in satisfactory agreement with those of other workers except in the case of Kussner, whose values for the aerodynamic coefficients seem to be systematically low.

G. Temple, England

2739. Weber, J., Theoretical load distribution on a wing with vertical plates, *Aero. Res. Counc. Lond. Rep. Mem.* 2960, 43 pp., 1956.

Whereas previous studies have been concerned with twin fin tail units, in which the fin height and tailplane span are of the same order, the present report relates to wing "fences" for which the height of the plate is usually less than a third of the wing span. The results are applicable directly to the spanwise load distribution on thin wings due to incompressible flow about configurations of minimum induced drag. They give approximate distributions for other planforms, including sweptback wings, and can be extended to subcritical flow regimes by the Prandtl-Glauert analogy.

For plates of equal height above and below the wings, a series of conformal transformations due to A. Betz is used. The starboard half of the *yz*-plane ($y > 0$) is transformed into the whole plane so that the wing remains a straight line and the plate becomes a parabolic arc. This arc is approximated by a circular arc, through its extreme and mid-points, and is then transformed into a full circle while the wing remains a straight line. Next, circle and straight line are transformed into a slit along the real axis, and, finally, into the right half plane. The spanwise load distribution is shown to depend only on two parameters which express the height of the plates and their spanwise location as fractions of wing semispan. For plates of small height situated inboard of the tip, the (vertical) distribution of side force is shown to be elliptic.

For plates on the upper surface only, the transformations are similar to those used previously by Mangler, and the parabolic arc is again approximated by a circular one.

Since the additional spanwise load distribution is similar for the two cases it is possible to interpolate for intermediate arrangements. It is claimed that the present theory offers a possible advance in the calculation of part-span vortex sheet effects. The mathematical theory of earlier pages is well supported by 27 figures which will be of interest and application to aerodynamicists and designers.

S. Kirkby, England

2740. Spreiter, J. R., and Sacks, A. H., A theoretical study of the aerodynamics of slender cruciform-wing arrangements and their wakes, *NACA Rep.* 1296, 31 pp., 1957.

[See AMR **9**, Rev. 3710].

2741. Eggers, A. J., Jr., Resnikoff, M. M., and Dennis, D. H., Bodies of revolution having minimum drag at high supersonic airspeeds, *NACA Rep.* 1306, 12 pp., 1957.

[See AMR **9**, Rev. 2594].

2742. Alksne, A. Y., Determination of vortex paths by series expansion technique with application to cruciform wings, *NACA Rep.* 1311, 13 pp., 1957.

[See AMR **9**, Rev. 3650].

2743. Roy, M., Principle of the study of jet wing (in French), *Rech. aéro.* no. 52, 3–11, July/Aug. 1956.

2744. Rodden, W. P., Surber, T. E., and Vetter, H. C., The effect of flexibility on the drag polar of an aircraft, *J. aero. Sci.* **24,** 6, 456–458 (Readers' Forum), June 1957.

2745. Morrison, W. D., Jr., and Alford, W. J., Jr., Effects of horizontal-tail position and a wing leading-edge modification consisting of a full-span flap and a partial-span chord-extension on the aerodynamic characteristics in pitch at high subsonic speeds of a model with a 45° sweptback wing, *NACA TN* 3952, 37 pp., June 1957.

2746. Tobak, M., On the minimization of airplane responses to random gusts, *NACA TN* 3290, 71 pp., Oct. 1957.

Author's summary abstracts the paper quite well. The gust alleviation treatment is exploratory in nature and needs further study to determine its limits and usefulness.

"A theoretical study is made of the motions experienced by aircraft in response to sharp-edged, harmonic, and random gusts. For the sharp-edged and harmonic gusts, exact responses in normal acceleration and pitching velocity are presented for the rectangular wing flying at Mach number 1.2. These are compared with approximate solutions based on commonly used assumptions, and the validity of each of the assumptions is assessed. It is determined that the use of stability derivatives in place of indicial functions in the equations of motion does not significantly impair the accuracy of solutions for transient and harmonic response.

"The problem of alleviating the airplane's response to random gusts is cast in a form amenable to treatment by the Wiener optimum filter theory. A derivation is given of the theoretical requirements of a compensating-force system that minimizes a linear combination of the airplane's mean-square normal acceleration and mean-square pitching velocity. Results of computations are presented which indicate the system may be successful in causing significant reductions of both motions."

M. G. Scherberg, USA

2747. Andrews, D. R., A flight investigation of the wake behind a Meteor aircraft, with some theoretical analysis, *Aero. Res. Counc. Lond. curr. Pap.* 282, 21 pp. + 12 figs., 1956.

An experimental investigation of the wake behind a Meteor 4 aircraft has been carried out, and theory used wherever possible to confirm and extend the results obtained.

Measurements show that for the conditions covered in these tests, the jet velocity has fallen to a negligible value by about 200–300 ft behind the jet exit. The jet would be expected to persist longest under take-off conditions, although no measurements were made of this specific case.

The major disturbances behind an aircraft are due to the trailing vortices, and these decay only slowly. Tests with a Vampire flying in the wake of the Meteor show that the strength of these vortices has only fallen to about half its initial value by 8000 ft behind the aircraft.

Theory and flight test experience show that the rolling moment imposed on a tracking aircraft constitutes the most severe disturbance from these vortices, and that in some circumstances this rolling moment can be sufficiently large to overpower the aileron control of the tracking aircraft.

Theory indicates that the disturbances from the trailing vortices will be very severe for a small slow aircraft flying in the wake of a large heavily loaded aircraft at low speed. The disturbances encountered by a missile attacking an aircraft will depend largely upon the relative sizes of the missile and aircraft, but in many cases may be small, as the closing speeds are usually high and the missile wing span small.

From author's summary

2748. Diederich, F. W., and Frischler, J. A., Jr., Unsteady-lift functions for penetration of traveling gusts and oblique blast waves, *Aero. Engng. Rev.* **16**, 8, 47–51, Aug. 1957.

The results of some calculations of the unsteady lift due to the penetration of sharp-edged traveling gusts are presented, and their use in such calculations as that of the lift due to the penetration by a given wing of an oblique blast wave or of the downwash field of another wing is discussed briefly.

From authors' summary

2749. Bergh, H., Experimental determination of the aerodynamic forces acting on oscillating wings (in Dutch), *Ingenieur* **69**, 23, L.7–L.11, June 1957.

A description is given of the measuring method used at the National Aeronautical Research Institute for determining the aerodynamic forces on oscillating wings. The test set-up and the testing procedure are considered in more detail. Results of the aerodynamic derivatives for a plain wing are dealt with.

From author's summary

2750. Yff, J., Measurements of some low speed oscillatory stability derivatives of the Agard wing model D *Nat. LuchtLab. Amsterdam Rap.* F. 195, 9 pp. + 8 figs., June 1956.

Some results of dynamic stability measurements of the Agard wing model D are presented together with a derivation of the equations of motion and a description of the test technique and the apparatus. The tests have been conducted in a low-speed wind tunnel at values of the reduced frequency ranging from 0.015 to 0.040 and at Reynolds numbers of 400,000–760,000.

It has been found that there is no appreciable effect of the reduced frequency on the aerodynamic derivatives. The results show good agreement with results of similar measurements which were conducted in France.

From author's summary

2751. Vandeperre, L. J., and Naar, J., The influence of wind upon a pole grillage (in French), *Ann. Trav. publics Belg.* no. 5, 5–52, 1956.

With the aid of known experimental data concerning air loads on truss pylons of equilateral triangular cross section, author develops semiempirical method for computing loads on separate faces of pylon. Air load coefficients are given as functions of angle of incidence for ratios of surface of truss members to total face surface between 0.1 and 0.4. Experimental verification of results for individual faces, which are important for structural design, is not available. In appendices, author gives summary of previous similar results for square cross section pylons, note on air loads on wires or cables, and some observations on velocities of strongest atmospheric air currents up to 500-m altitude in Western Europe and the Belgium Congo.

F. J. Plantema, Holland

2752. Parkus, H., Stability of a helicopter, *Jahrbuch Wissenschaft. Gesellsch. Luftfahrt*, 1956, 55–57.

The general equations for the motion of the helicopter are derived when elastic deformations are neglected under linearized flow conditions; such equations are in number of $n + 7$, and of $n + 4$ if motion in a vertical plane is considered (the n blade flapping motions, the rigid motion of the aircraft, the rotation of the rotor).

Simple kinematic considerations allow expressing the kinetic energy of the aircraft in terms of the parameters defining the $n + 4$ degrees of freedom. Lagrange's equations are then written; in the relationships so obtained the angle of rotation is substituted for the time as independent variable. The resulting equations are linear; the coefficients are periodic functions of the said angle of rotation, of period 2π.

If a Fourier series expression for the unknowns is set into the equations and integration from 0 to 2π is performed, a system with constant coefficient is obtained. This is allowed, however, if the period of oscillations is great as compared with that of rotation; otherwise the starting system is to be considered.

Nothing is said about the solution of the system, except that pure periodic solutions represent the borderline between stable and unstable solutions.

P. Santini, Italy

2753. Santangelo, G., Supersonic civil aircraft jet-propelled by turbojet with afterburner (in Italian), *Aerotecnica* **37**, 3, 133–146, June 1957.

After having shown that the future civil long-range transport aircraft is likely to be supersonic, a discussion is carried out as to lift systems and propulsion systems appropriate for high altitudes and high Mach number flight.

Author concludes that delta flying-wing with subsonic leading edges appears to be the most practical and efficient solution of the problem from an aerodynamic, structural, and flying-qualities point of view. The most efficient propulsion system is believed to be the conventional turbojet with afterburner, which, in the supersonic range, appears to be a really appropriate system from every point of view.

At take-off and landing the civil supersonic aircraft is very likely to make use of the lift obtained, totally or partially, through jets.

From author's summary

Aeroelasticity (Flutter, Divergence, etc.)

(See Revs. 2749, 2750)

Propellers, Fans, Turbines, Pumps, etc.

(See Revs. 2409, 2443, 2445, 2465, 2467, 2590, 2662, 2663, 2674, 2675, 2680, 2683, 2686, 2694, 2703, 2756, 2757, 2758, 2762, 2820, 2834, 2837, 2839)

Flow and Flight Test Techniques

(See also Revs. 2690, 2709, 2840)

2754. Hamilton, A. R., Extended range thermal conductivity vacuum gauge, *Rev. sci. Instrum.* **28**, 9, 693–695, Sept. 1957.

Two magnetic amplifier circuits for automatically controlling a constant-temperature hot-wire vacuum gage are described. The differential circuit senses temperature variations by means of voltage and current control coils which respond to the ratio of voltage to current in the hot wire. The bridge circuit senses temperature variations by unbalance in a Wheatstone bridge circuit. In either case, the sensing signal is amplified and fed back to the hot wire in such a direction as to very nearly restore the original temperature. Experimental data are given showing good gage sensitivity up to 50 mm. Circuitry is simple and, since the magnetic amplifier is inherently a stable device with no perishable components, long trouble-free life can be expected.

From author's summary

2755. Eisenberg, P., Craven, J. P., and Luistro, J. A., The David Taylor Model Basin micropressure range, *David W. Taylor Mod. Basin Rep.* 779, 25 pp., June 1957.

As a result of the pressure mine threat, model studies were initiated in 1940 for the measurement of ship and countermeasure

pressure fields. The requirements were such that an essentially permanent micropressure range was installed. This report describes the equipment which comprises the present range, discusses the relevant problems in measurement and operation, and cites alternate methods for the solution of each instrumentation problem.

Three types of transducers are described which operate on the capacitance, resistance, and inductance principles, respectively. The characteristics of transducers previously and currently employed at the Model Basin are also presented. The reference pressure system, the automatic balancing system, and the remote calibration system are described in detail and the electronic components are described in brief.

Extraneous problems in calibration and operation are discussed and a sample nonclassified record is presented.

From authors' summary

2756. Kling, R., Application of microphotography of fuel sprays to the study of jet engine combustion chambers (in French), Paper presented at Colloquium on flow and combustion, Freudenstadt, 25 pp. + 11 figs. + 15 ref., 1957.

The spray produced by Duples-type nozzles in the combustion chamber of a Nene engine was studied by means of photomicrographs. Spark illumination produced by a Frungel-strobolamp (made by Dr. F. Frungel, G. m. b. H., Hamburg-Rissen) was used, using two sparks with a time interval of 100 microseconds. The sparks were initiated by the contactor of a robot camera, the shutter of which had an open interval of 1/25 sec. From the two successive pictures of the same droplet the velocity of droplet movement and its direction could be determined. The adaptation of the combustion chamber for this investigation by fitting it with quartz windows, the optical system and the mounting of the camera are described and illustrated in detail. The pictures taken were evaluated for Sauter mean diameter, and for a qualitative indication of the chamber turbulence. Viewing windows were provided at three distances from the injector, whereby it was made possible to study the change of the spray characteristics as the spray progresses along the axis of the combustion chamber.

K. J. De Juhasz, Germany

2757. Kling, R., and Leboeuf, R., An ultra-rapid microphotographic method and its application to the study of formation and development of combustible sprays (in French), Actes du Deuxieme Congres Internationale Photographie et Cinematographie Ultra-Rapide, 424–431 + 13 figs. + 5 ref., 1956.

Microphotography technique for studying sprays has to satisfy three main requirements: (1) a light source small enough and light duration short enough to produce an unblurred picture of the smallest droplet in motion; (2) an optical system to provide a sufficient distance between the lens and the spray to make possible the spray study without or with combustion; (3) it should be possible to take a large number of pictures in a short time interval for a study of motion and spray development, and of drop-size distribution. The apparatus developed by the authors comprises a pinpoint light source by a spark occurring between two tungsten electrodes in argon at 6 atm pressure, generated by the discharge of a 0.04 microfarad condenser charged to 6 kilovolts; the duration of flash is about 150 mm with a 1 : 1 magnification; this provides a visibility for a droplet of about 8 microns. The field of view is about 4 × 4 mm, and the photographic frame is 24 × 24 mm. The pictures are examined under a microscope; a droplet of 10 microns diam appears as a spot of 1-mm diam. With this equipment, two impinging liquid jets, the resulting liquid sheet, and its breakup into filaments and droplets were investigated. Photos of the spray are shown; graphs of droplet-size distribution in terms of total surface versus distance from impact point of the two jets are given.

K. J. De Juhasz, Germany

2758. Emery, J. C., Low-speed cascade investigation of compressor blades having loaded leading edges, *NACA TN* 4178, 76 pp., Jan. 1958.

Blade cambers having isolated-airfoil lift coefficients of 0.6, 1.2, 1.8, and 2.4 were tested over the usable angle-of-attack range for inlet-air angles of 30°, 45°, and 60° at solidities of 1.0 and 1.5. A comparative evaluation has been made of the results of this and previous cascade investigations of compressor blades having uniform chordwise loading as isolated airfoils and those having more loading at the trailing edge.

From author's summary

2759. Anscombe, A., and Williams, J., Some comments on high-lift testing in wind tunnels with particular reference to jet-blowing models, *J. roy. aero. Soc.* **61**, 560, 529–540, Aug. 1957.

Paper considers some of the special problems of wind-tunnel testing which arise in high-lift work. Discussion is confined to current experience in the R. A. E. and N. P. L. low-speed wind tunnels, and refers mainly to tests on blowing over flaps or jet-flaps. Comments are made on suitable size of model, methods of feeding compressed air into models without affecting balance readings, and general test technique. Methods of model construction are not specifically discussed.

From authors' summary

2760. Covert, E. E., Jet simulation in the wind tunnel, Mass. Inst. Technol., Nav. supersonic Lab., TN 252, v + 14 pp. + ref., July 1957.

The question of jet simulation in wind tunnels is discussed briefly. It is indicated that simulation in subsonic tests is not too difficult and that either hot or cold jets are satisfactory. In the subsonic case, the proper similitude parameter is the thrust coefficient. In supersonic tests, the problem is more difficult, but it appears that two suitable approximations can be made. If the field near the jet is of importance, the flow turning angle and the jet Mach number govern the flow. From author's summary

2761. Kunkel, W. B., and Hurlbut, F. C., Luminescent gas flow visualization for low density wind tunnels, *J. appl. Phys.* **28**, 8, 827–835, Aug. 1957.

At static pressures below a few millimeters of mercury the optical thickness of wind-tunnel gas streams becomes insufficient for conventional flow-visualization techniques in aerodynamic testing. In the University of California low-density wind tunnel, where the free-stream static pressure is of the order of one-tenth of one millimeter of mercury, use has been made of various gaseous afterglow phenomena in the visualization of low density flows. The present paper describes the successful application of afterglows in flows of nitrogen, air, argon, and helium.

On the basis of recent tests, the range of practicability of the enriched air-glow technique was found to extend from Mach 2 to Mach 4. The argon and helium glows were found to be useful in the range Mach 1.3 to Mach 2. Discussion of the ranges of applicability and of other aspects of the observed glows is supported by representative glow photographs.

From authors' summary

2762. Emrich, R. J., and Wheeler, D. B., Jr., Wall effects in shock tube flow, *Phys. of Fluids* **1**, 1, 14–23, Jan./Feb. 1958.

Measurements of pressure, density, and shock velocity were made to study deviations of real flow in shock tube from "ideal" flow, in which wall effects are neglected. Results are compared with theories of Trimpi and Cohen [AMR **8** (1955), Rev. 3508] and Mirels and Braun [AMR **10** (1957), Revs. 536 and 4152]. Trends of both density and pressure variations are predicted reasonably well by both these theories, in conditions where they might be expected

to apply, but there are disagreements in detailed behavior. Shock attenuation agrees moderately well with both theories, but agrees even better with simple exponential formula, with suitable choice of constant. Attention is drawn to importance of mixing which occurs at cold front, not taken into account in either of the theories. W. A. Mair, England

2763. Iwasaki, M., On the errors of a time counting system for use in experiments with a shock tube, *J. Japan Soc. aero. Engng.* **5,** 40, 128–133, May 1957.

A time counting system for use in a shock tube having an accuracy of 5 counts of 1 Mc/s pulses is made, and the process of improvement of its accuracy is described.

The development of detached shocks ahead of 180° and 10° wedge sections in the transonic region is experimented with the shock tube. From author's summary

2764. Glass, I. I., and Hall, J. G., Shock sphere—an apparatus for generating spherical flows, *J. appl. Phys.* **28,** 4, 424–425, Apr. 1957.

Some experimental results are presented of the flow generated by the explosion of a sphere of high-pressure gas. Glass spheres of 1, 2, and 5-in. diam containing air, He, or SF_6 at overpressures up to 21 atmos were used to produce a spherical analog of the shock tube. The results show that the shock sphere may be applied to the study of explosions, implosions, and spherical wave interaction under controlled and known initial conditions. From authors' summary

2765. Powell, J. B. L., The diffraction of a rarefaction wave by a corner, *J. Fluid Mech.* **3,** 3, 243–254, Dec. 1957.

Investigation is made of the effect of a small disturbance on the flow in a complete rarefaction wave (i.e. flow produced when a membrane separating a compressible gas from a vacuum is ruptured, or a piston is retracted at the so-called escape speed of the gas). The perturbation arises from a rigid boundary slightly inclined to the direction of flow. The growth of the perturbed region is studied and the pressure field is calculated for diatomic gases. General solution of basic equations and the nature of the expanding boundary of the perturbed region are investigated for variable adiabatic index. In later chapters are treated the velocity and pressure fields at the wall surface, and the position of the sonic line. This thorough investigation, suggested by Dr. W. Chester, was made because, in practice, it is impossible to obtain a perfect vacuum behind the membrane. Shock-tube experiments are usually performed with the pressure difference across the membrane obtained by a high compression of the gas on one side, consequently the resulting flow differs from the complete rarefaction wave. J. J. Polivka, USA

2766. McGregor, D. M., An experimental investigation of the oscillating pressures on a circular cylinder in a fluid stream, *Univ. Toronto Inst. Aerophys. UTIA TN* 14, 16 pp. + 21 figs., June 1957.

Results of experimental investigations of the fluctuating pressures at the surface of a circular cylinder are presented. The cylinder was mounted in the UTIA subsonic wind tunnel at right angles to the flow and contained within it the microphone used as the pressure transducer. The pressure level of the fundamental shedding frequency was very low at the front of the cylinder, but rose sharply to a large maximum at the sides. After remaining constant over an appreciable arc, it decreased to the background level at the rear. The level of the second harmonic was masked by the background except in the rear 1/3 of the cylinder where it increased steadily to a maximum in the downstream direction. An attempt to develop a simple mathematical model of the flow by considering an alternating vortex standing at the rear of the cylinder met with moderate success. Estimations were made of the fluctuating lift and drag coefficients of the cylinder. From author's summary

2767. Johnson, J. L., Jr., Wind-tunnel investigation of the static longitudinal stability and trim characteristics of a sweptback-wing jet-transport model equipped with an external-flow jet-augmented flap, *NACA TN* 4177, 89 pp., Jan. 1958.

The investigation included tests of the model to determine the effects of wing position, vertical position of the horizontal tail, and size of the horizontal tail for a range of momentum coefficients up to about 4. The model had a 30° sweptback wing with an aspect ratio of 6.60. Calculations were made to compare the relative merits of various trim devices for use on airplanes equipped with jet-augmented flaps. From author's summary

2768. Zalovcik, J. A., Calculated effect of some airplane handling techniques on the ground-run distance in landing on slippery runways, *NACA TN* 4058, 12 pp. + 2 tables + 16 figs., July 1957.

Some calculations were made on the basis of simplifying assumptions to determine the effect on the ground-run distance of maintaining a nose-high attitude instead of a three-point attitude in landings of several types of jet airplanes on slippery runways. The airplanes considered were a sweptwing transport and unswept-, swept-, and delta-wing fighters. The effect of such factors as speed, braking effectiveness, and residual thrust on the difference in ground-run distance with the two handling techniques is briefly considered. Some computations were also made to indicate the effect of instantaneous flap retraction on the ground-run distance. From author's summary

2769. Schade, R. O., Flight-test investigation on the Langley control-line facility of a model of a propeller-driven tail-sitter-type vertical-take-off airplane with delta wing during rapid transitions, *NACA TN* 4070, 19 pp., Aug. 1957.

A flight-test investigation has been made on the Langley control-line facility to determine the longitudinal stability and control characteristics of a model of a propeller-driven tail-sitter-type vertical-take-off airplane with delta wing during rapid transitions from hovering flight to forward flight and back to hovering. The control-line facility provides for the flying of models in a large-diameter circle by means of a control-line technique similar to that used by model-airplane enthusiasts. The present investigation showed that the facility was generally satisfactory for investigating the characteristics of vertical-take-off models during rapid transitions. It was found that rapid transitions from hovering flight to forward flight could be performed fairly easily, but precise longitudinal control was necessary to perform the transitions smoothly. The transitions from forward flight to hovering flight were more difficult to perform because there was a greater variation of power settings which require closer coordination of the power and pitch control. From author's summary

2770. Buell, D. A., and Tinling, B. E., Ground effects on the longitudinal characteristics of two models with wings having low aspect ratio and pointed tips, *NACA TN* 4044, 16 pp. + 2 tables + 21 figs., July 1957.

The ground effects on the longitudinal characteristics of two models with wings having low aspect ratio and pointed tips have been determined from wind-tunnel tests at Reynolds numbers from 2.5 to 10 million, using a flat plate to represent the ground. The first model had an aspect ratio of 2 and used trailing-edge flaps for longitudinal control. The flap hinge line had no sweepback and the flap chord was 25% of the wing chord. The second model

had a triangular planform of aspect ratio 3 and was equipped with flaps and a conventional tail.

The test results showed that the presence of the ground increased the lift-curve slope, decreased the drag due to lift, and increased the stick-fixed stability of the models. The latter effect was most pronounced on the model with the horizontal tail. The ground effect on the control-surface deflections for balance was small on the tailless model but was sizable on the tailed model. Control-surface hinge moments, measured only on the tailless model, were little affected at a given lift coefficient.

The experimentally determined ground effects on the lift and drag characteristics were generally underestimated by the theory of Tani, et al., at the higher lifts. When applied to the estimation of ground effects on the variation of pitching moment with lift coefficient of the tailed model, the theory had errors which tended to be compensating. From authors' summary

Thermodynamics

(See Revs. 2406, 2443, 2677, 2688, 2689, 2700, 2701, 2777, 2794, 2799, 2812, 2821, 2891, 2894, 2895)

Heat and Mass Transfer

(See also Revs. 2441, 2637, 2730, 2731, 2820, 2852, 2881, 2882)

2771. Anderson, O. L., Cooling time of strong glass fibers, *J. appl. Phys.* **29**, 1, 9–12, Jan. 1958.

Quenching time of fine glass fibers spun in air is calculated by the standard relations to be of the order of 10^{-4} seconds, based on the Nusselt number for heat removal being less than unity. Quenching time is comparable to the Maxwell relaxation time, but an order of magnitude greater than the heat transfer lag.

This quick quenching must inhibit flaw formation, which explains the high strength of such glass fibers.

C. F. Bonilla, USA

2772. Yang, K.-T., Transient conduction in a semi-infinite solid with variable thermal conductivity, *J. appl. Mech.* **25**, 1, 146–147, Mar. 1958.

Paper is an analytical treatment of the classical problem of transient conduction in a semi-infinite solid when its surface undergoes a stepwise temperature change and the variation of thermal conductivity with temperature is allowed.

W. L. Sibbitt, USA

2773. Colluro, G. B., The order of layers in heat insulation (in Italian), *Termotecnica* **11**, 10, 481–488, Oct. 1957.

Author presents a study on the thermal insulation in a wall composed of *n* different slabs of finite thickness considering the influence of the order of the layers. Author studies the case of steady flow and the case of the time-variable surface conditions, using the method of the analogy with a problem on electric current. He proves that, in general, the order of the layers influences the rate of the heat absorbed by the wall, except when the product *cdk* (*c* specific heat, *d* density, *k* thermal conductivity) may be constant for all the different slabs. Some numerical valuations are given. G. Sestini, Italy

2774. Ratnikov, V. F., Calculation of the heat concentration in furnace walls (in Russian), *Trudi Uralsk. politekhn. in-ta* no. 53, 118–132, 1955; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7566.

A simplified method is presented for determining the quantity of heat absorbed by furnace walls in unsteady working conditions. Applying the method of hydrothermal analogs, author has simulated, in application to the unidimensional problem, the temperature fields and heat flows for different initial and boundary conditions.

Investigation of the influence of intensity of heat exchange with the ambient medium on the total heat absorption of the wall with an instantaneous temperature rise on the inner surface to 1000°, and subsequent prolonged holding at this temperature, leads to the conclusion that the value of the coefficient of heat loss from the outer surface of the wall to the ambient medium practically affects the total heat absorption at Fourier numbers of and over $F_0 \geqslant 0.47$.

It is further noted that formulas referring to infinitely thick walls give, for Fourier number of $F_0 \leqslant 0.6$–0.65, a sufficiently accurate determination of the total heat absorption for practical purposes.

The results of comparing the heat flows indicate that for the case where the temperature of the inner surface of the wall increases linearly, and continues to be maintained at the maximum value attained, the quantity of heat absorbed by the wall differs little from the heat absorption in the case of instantaneous heating of the inner surface to the same end temperature, subsequently held for a time equal to the sum of the holding period at steady temperature and half the period of heating with a linear temperature rise.

The divergence of the calculated values of the heat flows does not exceed 6% in the upper limit.

Results are given of simulating the temperature fields in single and double walls.

To calculate the heat absorption of a single wall, curves are reproduced expressing the relationship between the nondimensional heat flow and the parameters

$$F_0 = \frac{\alpha\tau}{S^2},\ B_i = \frac{\alpha_0 S}{\lambda}$$

The experiments with double walls did not produce any generally valid relationships. Each calculation chart presented by the author has been obtained for particular values of the coefficient of heat loss from the outer wall surface (α_0) and the ratio of the thicknesses of the two parts of the wall (S_1/S_2).

The total heat absorption of a compound (double) wall, its nondimensional form, can be represented as a fraction of F_0 and the ratio of the coefficients of temperature conductivity of both parts of the wall.

The article contains numerical examples, demonstrating the process of calculation and the utilization of the charts supplied.

There are typographical errors.

M. D. Vaisman

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2775. Thomas, P. H., Some conduction problems in the heating of small areas on large solids, *Quart. J. Mech. appl. Math.* **10**, 4, 482–493, Nov. 1957.

This paper derives mathematically a number of results for the heating of small parts of large solids or thin sheets. Author considers first the heating of a circular area on the surface of a semi-infinite solid, and determines in particular the transient and steady-state temperatures at the center of the heated area. Next he considers heating of an infinite thin sheet over a circular area, and then over an infinite strip. Results are derived analytically, and in some cases are represented graphically as well. A table summarizes results of importance to those interested in fire damage. R. E. Gaskell, USA

2776. Paschkis, V., and Hlinka, J. W., Rate of temperature change of simple shapes, ASME Ann. Meet., New York, N. Y., Nov. 1956, Pap. 56-A-118, 7 pp.

Paper deals with the rate of temperature change and its calculation. Charts have been developed to determine such rates for the three simple shapes: the slab, cylinder, and sphere. In each case the shape is originally at constant temperature, and the surrounding ambient is itself undergoing a step change of temperature. Also given are a list of the available charts for this problem and suggestions for other problems related to such charts.

A. Pignedoli, Italy

2777. Powell, R. W., Experiments using a simple thermal comparator for measurement of thermal conductivity, surface roughness and thickness of foils or of surface deposits, *J. sci. Instrum.* **34,** 12, 485-492, Dec. 1957.

This thermal comparator consists of two metal balls similarly mounted in a block of balsa wood, but one is at a slightly lower level so that it touches any surface on which the block rests. After heating to a small fixed temperature excess, the block is laid in contact with the test surface. Differentially connected thermocouples attached to each ball measure the rate of cooling of the ball which makes contact. The initial rate of cooling is function of the thermal conductivity of the material and of the condition of the surface. Calibration is made with at least two substances of known thermal conductivity and of comparable surface finish. Accuracy in thermal conductivity measurements is about ± 6%, but lead differs considerably, due to its softness. Alternatively, the device can be used as a means of measuring surface finish and thickness of surface deposits.

From author's summary by A. E. Brun, France

2778. Morgan, W. R., and Baxter, W. G., Thermal conductivity of insulating materials for use in nuclear reactors, ASME Fall Meet., Hartford, Conn., Sept. 1957. Pap. 57-F-9, 5 pp.

Paper presents the results of thermal conductivity measurements of two relatively high-temperature insulation materials, both under nuclear radiation and in the absence of nuclear radiation. The apparatus employed for the measurements is described. Thermal conductivity results are presented graphically and some conclusions pertaining to particle bombardment are given.

From authors' summary

2779. Reichenbach, G. S., Experimental measurement of metal-cutting temperature distributions, ASME Semiann. Meet., San Francisco, Calif., June 1957. Pap. 57-SA-53, 15 pp.

Two new methods of measuring cutting temperatures have been evaluated. The first is a radiation technique using a lead-sulphide cell. The cell is arranged to sight through a small hole drilled in the work material sensing radiation from the shear plane and clearance face of the tool. Peak temperatures in these regions have been obtained, but shear-plane temperature distributions could not be determined successfully. The second technique uses a 0.005-in. single wire imbedded in the side of a workpiece as a thermocouple. With orthogonal cutting the tool passed beneath the wire so that the wire passed up the tool face still imbedded in the chip. By this method it was possible to determine the temperature field throughout the chip and work. These temperature fields have been compared to theoretical predictions and show only fair agreement. The lack of agreement is due partially to the simplifying assumptions of orthogonal cutting and a true geometric shear plane necessary for theoretical solution. The necessity of locating the thermocouple junction on the side of the work introduced errors from side flow of the chip and certain other edge effects. The iteration procedure of Trigger and Chao has been extended to tools of any rake angle and simplified by use of a conducting-paper electrical analog.

From author's summary

2780. Kececioglu, D., Shear-zone temperature in metal cutting and its effects on shear-flow stress, ASME Semiann. Meet., San Francisco, Calif., June 1957. Pap. 57-SA-70, 7 pp.

Relationships are given from which the mean shear-zone temperature in oblique, as well as in orthogonal, cutting can be calculated. The mean shear-zone temperature, developed when machining SAE 1015, 118 Bhn seamless-steel tubing under a wide range of cutting conditions, is calculated and is found to vary from about 410 to 840 F. The effect of the mean shear-zone temperature on the mean shear-flow stress is studied. In general, the mean shear-flow stress is found to decrease by about 1800 psi for a 100-F increase in the mean shear-zone temperature within the range of 410 and 840 F. This compares with the little or no effect of the mean shear-zone temperature on the mean shear-flow stress reported by Shaw and Finnie and Chao and Bisacre. A more comprehensive approach is proposed to establish the effect of the important shear-zone factors on the mean shear-flow stress.

From author's summary

2781. Morton, B. R., On the equilibrium of a stratified layer of fluid, *Quart. J. Mech. appl. Math.* **10,** 4, 433-447, Nov. 1957.

A mathematical study shows the importance of viscosity and thermal conductivity (or other properties concerning diffusion) on departure from equilibrium of continuously stratified layers of fluid. The initial rate of growth of small disturbances is shown to depend on the Rayleigh and Prandtl numbers. The effect of nonuniform stratification is also investigated and is found to have little effect. The general treatment is idealized for mathematical purposes, but real systems should be qualitatively similar.

J. M. DallaValle, USA

2782. Lynn, S., Corcoran, W. H., and Sage, B. H., Material transport in turbulent gas streams: radial diffusion in a circular conduit, *AIChE J.* **3,** 1, 11-15, Mar. 1957.

Experimental studies of the mixing of coaxial streams of natural gas and air were made at Reynolds numbers of 44,000 and 79,000. The turbulent-velocity profile was altered as little as feasible by the central injection of the air stream. Diffusion coefficients of mass transport and momentum were calculated at various axial positions along the test section.

The results are limited to this apparatus because the flow was nonuniform within the test section. The nonuniformity of the flow resulted from asymmetrical entry of the gas in the entrance section, from an abbreviated length of entrance section, from a boundary discontinuity at the point of air injection, and from a boundary discontinuity at the beginning of the test section.

Reviewer believes that the numerical factor 8 in Eq. (9) should be 4. Reviewer also believes that the use of a uniform-flow relationship of Eq. (10) in the calculation of the linear-momentum diffusion coefficient is subject to question in the nonuniform flow zone. The method of computation of diffusion coefficients is of interest.

M. R. Carstens, USA

2783. Metzner, A. B., Vaughn, R. D., and Houghton, G. L., Heat transfer to non-Newtonian fluids, *AIChE J.* **3,** 1, 92-100, Mar. 1957.

Authors analyze non-Newtonian flow for both pseudo-plastic and dilatant fluids and develop methods for relating heat transfer in such fluids to that for Newtonian fluids in terms of a flow behavior index. The resultant expressions are similar to those suggested for Newtonian fluids by McAdams. Experimental data on four non-Newtonian fluids in the laminar flow region are correlated by these equations with a mean deviation of 13.5%. Preliminary correlation is also given for a small amount of data at the low Reynolds number and of the turbulent region.

M. A. Mayers, USA

2784. Kirpikov, V. A., Heat transfer in helically coiled pipes (in Russian), *Trudi Moskov. Inst. Khim. Mashinostrojenija* **12,** 43–56, Moscow, 1957.

Author investigated experimentally the local and average heat transfer coefficients for water flowing in helically coiled copper pipes for Reynolds numbers of $10^4 - 4.5 \times 10^4$. The coils were heated by condensing steam. Main geometric parameter studied was the ratio (coil radius r/tube inner diameter d). Results were presented in the form

$$N = \text{constant } R^{0.8} P^{0.4} \left(\frac{d}{r}\right)^m$$

and an appreciable increase of N with reduced r/d was observed. Correlation of results ignored the variation of the ratios of (coil length/d) and of (coil pitch/d).

Author also measured average friction coefficients in isothermal and nonisothermal flow and correlated these in the form

$$\xi = \frac{\text{constant}}{R^p} \left(\frac{d}{r}\right)^q.$$

The data are incomplete because the correlation again ignores the effects of length and pitch, and also because, for nonisothermal flow, the temperature ratio between wall and fluid does not appear in the correlation.

One conclusion that can be drawn from the data on local heat-transfer coefficients is that the entrance length effect extends over a longer length than for straight tubes. Unfortunately, the validity of this and some other conclusions is in doubt, because the coils had inlet and outlet pipes which were co-axial with the coils, involving sharp bends at each end. Tangential inlet and outlet pipes should have been used.

Y. R. Mayhew, England

2785. Kays, W. M., and Bjorklund, I. S., Heat transfer from a rotating cylinder with and without cross flow, ASME Ann. Meet., New York, N. Y., Nov. 1956, Pap-56-A-71, 8 pp.

Authors have studied the effect of forced flow normal to a horizontal cylinder, heretofore neglected. A relation for the Nusselt number is established for the no convection case, and excellent experimental agreement at higher rotating Reynolds numbers is shown where the convection Reynolds numbers are negligible. This relation is based upon the von Karman analogy of heat and momentum transfer in a laminar sublayer and "buffer layer". Temperature measurements made in these layers with natural convection agree fairly well with those predicted from the analogy. It is noted that the deviation in the laminar sublayer is greater for the higher rotating Reynolds number, attributable perhaps to the higher shearing rates. Since the laminar sublayer is thinner at higher rotating speeds, the deviation may also be due to experimental error.

Experimental data presented indicate that at low rotating speeds the cross flow is the controlling factor. As the rotating speed is increased the cross flow has less influence and finally becomes negligible. The curves show a well-defined knee where these controlling factors form two different regimes.

As an extention of the work presented, it would be of interest to see data for the rotating cylinder oriented in a vertical direction both with natural convection and forced flow parallel to the cylinder axis. It is suspected that the transition between these regimes will no longer be well defined.

H. Merte, Jr., USA

2786. Topper, L., Forced heat convection in pipes, *Indust. Engng. Chem.* **48,** 8, 1379–1382, Aug. 1956.

Author presents a steady-state solution of the forced-convection heat equation assuming temperature to be independent of fluid properties. Solution is applicable to heating of a fluid flowing through a tube under plug flow conditions and for constant wall heat flux; it also applies, with modification, to turbulent flow and analogously to certain zero-order heterogeneous chemical processes in tubular reactors under similar boundary conditions. The solution, valid within entrance region and downstream, gives temperature located radially and axially as a function of tube radius, fluid thermal properties, velocity, and wall heat flux.

Author indicates that the validity of the assumption that axial temperature gradient is uniform with respect to radius and equal to the mixed-mean axial gradient made by Jacob and Rees, and Poppendiek, is a function of the inverse Prandtl-Reynolds product and the distance downstream.

S. K. Fenster, USA

2787. Gadd, G. E., A review of theoretical work relevant to the problem of heat transfer effects on laminar separation, *Aero. Res. Counc. Lond. curr. Pap.* 331, 4 pp. + 2 figs., 1957.

The results of various theories concerning the effects of heat transfer on laminar separation are briefly discussed. All the theories agree in predicting that wall temperature has an important influence on separation conditions.

From author's summary

2788. Nicollian, E. H., Gunther-Mahr, G. R., and Weisberg, L. R., A radiant-energy heater using an ellipsoidal reflector, *IBM J. Res. Devel.* 1, 4, 349–355, Oct. 1957.

The effectiveness of a radiant-energy heater employing a hemi-ellipsoidal aluminum reflector and an incandescent lamp as radiation source is illustrated by application to the zone-melting of germanium. The factors affecting the design and performance of this heater are discussed.

From authors' summary

2789. Irvine, T. F., Jr., Hartnett, J. P., and Eckert, E. R. G., Total normal emissivity measurements for porous materials used for mass-transfer cooling, ASME Fall Meet., Hartford, Conn., Sept. 1957. Pap. 57-F-8, 5 pp.

Total normal emissivity data are given for six commercial-type stainless-steel porous surfaces, in the temperature range 200 to 600 F. Comparisons are given with original material, and with absorptivity for solar radiation.

A. Whillier, South Africa

2790. Bruges, E. A., Performance of heat exchangers, *Engineer, Lond.* **204,** 5299, p. 225, Aug. 1957.

Paper advocates the use of the ratio of the available energy transfer as criterion for assessing performance of heat exchangers. Illustrations are given to show how the pressure drop in the fluids can then be taken into account. Heat losses can also be allowed for. Author discusses the effect of the dependence of specific heat on temperature on the possibility of obtaining 100% effectiveness under certain conditions.

Paper raises a number of interesting points, though these may be found academic to most normal designers and users of heat exchangers. The relevance of this paper to cases where the effecttiveness approaches 100% should, however, be considered. Reviewer disagrees with definition of effectiveness employed and would rather it was defined as the ratio of heat received by the heated fluid to that which the heated fluid would receive if its temperature was raised to the inlet temperature of the heating fluid. The effectiveness (which, it is agreed, is not equivalent to efficiency) is then a straight ratio of temperature differentials in all cases.

G. G. Thurlow, England

2791. Kourim, G., Circuit diagrams for analog simulating heat flow in nuclear reactors (in German), Archiv fur Technisches Messen und Industrielle Messtechnik, Leaflet no. 263, Dec. 1957.

Two-dimensional circuit is shown by simplifying cross section to a system of circular symmetry around a single fuel rod and considering axis as second dimension. Radially, each element (fuel element, coolant, tube wall, and moderator) is represented by a single node. Heat exchange with flowing coolant makes use of a (non-described) ideal cathode follower, with infinite input impedance and zero output impedance. Perimeter of system is fully insulated. A second analogous system is suggested in which temperatures are represented by currents, rates of heat flow by voltages, thermal conductance by electric resistances, and heat capacitance by inductances. Such a network could be tied directly to a simulator for the neutron flux of the system, in which the heat output appears as voltage. Reference is made to author's dissertation, Stuttgart 1956 and to Kourim, G., *Regelungstechnik* **5**, 163-167, 1957. V. Pachkis, USA

2792. Juhasz, S., and Clark, J., Hydraulic analogy for transient conditions in heat exchangers, IXe Congres International de Mecanique Appliquee, Actes, II, Universite Bruxelles, 495-505, 1957.

Paper gives a full description of a hydraulic analogy for studying transient conditions in heat exchangers with particular respect to the transient parallel-flow and counterflow heat exchanger. A full mathematical treatment is included and the thorough presentation makes this a valuable report. G. G. Thurlow, England

2793. Juhasz, S., Hydraulic analogy for transient crossflow heat exchangers, ASME Ann. Meet., New York, N. Y., Dec. 1957. Pap. 57-A-125, 11 pp.

Paper extends the application of a hydraulic analog, already described by author and co-workers for various simpler heat-exchanger systems, to the case of a one-pass transient crossflow heat exchanger. It includes a useful survey of the application of analog techniques to this type of problem, with valuable bibliography. The new analog proposed involves, as previously, the flow of liquid to represent the flow of heat. The liquid level in the various containers representing parts of the hot fluid, the heat-transfer material or the cold fluid corresponds to the temperature of that component, specific heat being allowed for in the cross-sectional area of the column. Heat flow between the various components is represented by fluid flow through capillaries, and the transient nature of the process is simulated by adjusting several rotary valves in a predetermined sequence.

This design can allow for variations in specific heat and heat conduction and convection coefficients. It has the advantage of making visible the "heat flow", and the analog should have educational value. No illustrations or examples of the use of the analog are given in the paper, however, and it would be valuable to know if the design performs well in practice.

The extensions of the analogy to multipass crossflow systems and the possibilities of using an electrical rather than a hydraulic system are outlined in this most comprehensive paper.

G. G. Thurlow, England

2794. Pozin, M. E., The efficiency of chemical equipment as a function of the material flow rates and of the rate coefficients of chemical, diffusion, and thermal processes, *J. appl. Chem., USSR* **29**, 9, 1439-1449, Sept. 1956 (Consultants' Bureau Translation, *Zh. Prikl. Khimy* **29**, 9, 1336-1346).

Paper presents the results of an analytic investigation relating the production rate and efficiency of equipment used in chemical, diffusional and thermal processes carried out under various conditions in process industries. The paper analyzes chemical processes of the first and second order for both batch and continuous types of flow. Heat-transfer and mass-transfer equipment for parallel flow, counterflow, and crossflow are studied. Generalized equations are given for the efficiency and equipment size for each of these cases and it is suggested that such equations can serve as the basis for the selection of optimum equipment. The paper gives no development of the relations reported and only indicates the final results for each of the cases covered. Only two references are cited, which are author's own works.

Y. S. Touloudian, USA

2795. Kuzminykh, I. N., and Rodionav, Mass transfer at sieve plates set at different slopes, *J. appl. Chem., USSR* **29**, 9, 1433-1437, Sept. 1956 (Consultants' Bureau Translation *Zh. Prikl. Khimy* **29**, 9, 1330-1335).

Paper presents experimental data on mass-transfer coefficients, for both the liquid and gas phases, in a rectifying column equipped with perforated plates. The experimental fluids consist of water and air flowing in counter-current direction. The perforated sieve plates are equipped with holes ranging from two to six millimeter in diameter and hole spacings of seven to twenty-one millimeters. These experimental sieve plates represented a free flow area of 7.4 to 20.4%. The main parameters under investigation were: (1) slope of the perforated sieve plates, ranging from 0 to 5° angle from the horizontal; (2) liquid mass flow rate; (3) vapor velocity. Data for various conditions of operation are reported in eight separate figures, each including a number of parameters. Unlike most Russian publications, authors report their raw experimental data as well as the smoothed curves fitting these data. Simple correlating expressions are presented; for their derivations, authors refer to five of their previous publications which are cited at the end of the paper. For the type of experiments reported, the precision of the data seems to be better than average; however, it is difficult to evaluate the accuracy of the reported results since the authors do not discuss the details of their experimental procedure nor their measurement techniques.

Y. S. Touloukian, USA

2796. Rychkov, A. I., and Planovsky, A. N., An analytical equation for determining the coefficient of heat loss in the boiling of liquids (in Russian), *Khim. prom-st.* no. 5, 31-34, 1955; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7571.

Applying the method of approximate simulation, authors establish the relationship $E_s = f(q/q_{cr}, p/p_{cr})$ where q/q_{cr} = ratio of actual to critical heat loading, characterizing the hydrodynamics of the system; p/p_{cr} = referred pressure, characterizing the thermophysical properties of the boiling liquid, E_s = a complex, non-dimensional factor designated the ebullioscopic coefficient, representing the product of the coefficient of heat loss α by the ebullition constant E of the boiling liquid, $E = RT^2/r$, where R is the gas constant, r the latent heat of vaporization, and T the absolute boiling temperature of the liquid.

The values for q_{cr} have been taken by the authors from the chart of Chickelli and Bonilla [M. F. Chichelli and C. F. Bonilla; *Trans. Amer. Inst. chem. Engrs.* **44**, p. 755, 1945]. From evaluation of the experimental results obtained by other research workers workers, authors deduce the relationship

$$E_s = K\left(\frac{q}{q_{cr}}\right)^{-0.28}, \text{ for } \frac{p}{p_{cr}} = \text{const}$$

and recommend for $p/P_{cr} < 0.35$, the formula

$$E_s = 0.19\left(\frac{q}{q_{cr}}\right)^{-0.28}\left(\frac{p}{p_{cr}}\right)^{-0.33}$$

P. I. Povarnin

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2797. Foster, H. H., and Ingebo, R. D., Evaporation of JP-5 fuel sprays in air streams, *NACA RM* E55KO2, 26 pp. + 10 figs., Feb. 1956.

Paper describes a continuous sampling-probe technique to determine the percentage of JP-5 fuel spray evaporated under conditions similar to those in ramjet engines. Fuel was injected contrastream from a multiple-orifice injector, and sampling data were obtained at distances of 7 to 18 inches downstream of the injector. Inlet-air conditions were varied over the following ranges: air velocities, 138 to 370 fps; air static pressure, 15 to 35 inches of mercury abs; and air temperatures, 80 to 700F. A formula has been developed to express the percentage of the JP-5 fuel spray evaporated as a function of the distance downstream from the injector, the heat-transfer driving force, the air velocity, and the fuel-injection velocity. The JP-5 fuel has a lower volatility (therefore is safer) than has the JP-4 fuel, but for the same reason the performance may be inferior. This investigation on evaporation served to supply data for the combustor design, to adapt it for the characteristics of the low-volatility fuel. Apparatus comprising fuel and sampling system, procedure, and method of analysis are described. Sample calculation of fuel-spray evaporation is given.

K. J. DeJuhasz, Germany

2798. Moeller, C. E., Resistance welding of platinum-10 per cent rhodium wire to platinum sheet to produce a well-defined thermocouple junction, ASME Ann. Meet., New York, N. Y., Dec. 1957. Pap. 57-A-118, 9 pp.

To measure temperatures of a sheet of platinum when exposed to high-velocity gases, a thermocouple was designed such that its thermal junction is well defined and in the form of a plane, parallel to and a known distance from the exposed surface. The junction is at the interface of the platinum sheet and a head of platinum 10 per cent rhodium, 0.040 diam and 0.007 in. thick on the end of a 0.010-in.-diam lead wire. It is mechanically produced by resistance welding, the optimum welding parameters being determined by metallographic examination of the weld specimens. The generation of heat at the weld surfaces stress-relieves the deformed platinum crystals of the cold-worked sheet. Thus the relative magnitudes of the heating effect can be determined by the size of the region of the annealed platinum crystals in the polished and etched cross section of the weld specimen. Performance data currently available consist of the static calibration of a number of the thermocouples. Results from the Diamond Ordnance Fuze Laboratories, Washington, D. C., indicate reproducibility of the elements within 3.5 μ volts at 960 C.

From author's summary

2799. Scheller, W. A., and Brown, G. M., The Ranque-Hilsch vortex tube, *Indust. Engng. Chem.* **49,** 6, 1013-1016, June 1957.

The Ranque-Hilsch tube has been the subject of interesting investigations in which attempts have been made to explain operation on the basis of terminal conditions or partial data. The object of this work was to measure pressure, velocity, and temperature profiles within the tube and deduce how it functions on the basis of these experimental data and the principles of fluid mechanics.

From authors' summary

Thirteenth annual unit operations review, *Indust. Engng. Chem.* **50,** 3, Part II, Mar. 1958:

2800. Flood, J. E., Jr., Centrifugation, 428-429.

2801. Bagnoll, E., Drying, 435-437.

2802. Badger, W. L., and Lindsay, R. A., Evaporation, 438-441.

2803. Weintraub, M., Flow of fluids, 447-451.

2804. Rushton, J. H., Mixing, 478-480.

2805. Work, L. T., Size reduction, 481-484.

Sixth annual fundamentals of chemical engineering review, *Indust. Engng. Chem.* **50,** 3, Part II, Mar. 1958:

2806. Johnson, E. J., Molecular transport properties of fluids, 488-491.

2807. Rose, A., Sweeny, R. F., Johnson, R. C., Burggrabe, W. F., and Heiny, R. L., Computer, mathematics, and statistics, 512-519.

2808. Williams, T. J., Process control and automation, 520-524.

2809. Oppenheim, A. K., Coldren, C. L., Grossman, L. M., and Sterling, C. V., Fluid dynamics, 525-542.

2810. Eckert, E. R. G., Hartnett, J. P., and Irvine, T. F., Jr., Heat transfer, 543-554.

2811. Wilke, C. R., and Prausnitz, J. M., Mass transfer, 555-560.

2812. Smith, J. M., and Brown, G. M., Thermodynamics, 561-568.

Combustion

(See also Revs. 2756, 2757)

2813. Fine, B., Stability limits and burning velocities for some laminar and turbulent propane and hydrogen flames at reduced pressure, *NACA TN* 4031, 49 pp., Aug. 1957.

The effect of reduced pressure on blowoff, flashback, and burning velocities of propane-oxygen-nitrogen burner flames was studied (oxygen fraction of oxidant, 0.5). The pressure exponent of burning velocity, 0.22, was nearly the same as for hydrogen-air flames; stability loops showed the same blowoff and flashback characteristics as were previously observed for hydrogen-air flames. In particular, for both systems, quenching distances determined as a function of pressure from the points of intersection of flashback and blowoff portions of stability loops were considerably higher than those obtained previously by a stopped-flow method.

Of the two systems, the hydrogen-air system showed larger burning velocity, greater stability toward reduced pressure, and higher reaction order, as calculated from a simple thermal equation, and the propane-oxygen-nitrogen system showed the larger reactivity based on flashback. For both systems, laminar and turbulent flashback followed the velocity-gradient concept. However, turbulent blowoff was successfully treated by a critical boundary-velocity gradient for propane-oxygen-nitrogen flames, whereas for hydrogen-air flames neither laminar nor turbulent blowoff conformed to the velocity-gradient principle.

Laminar flashback was studied for hydrogen-argon-"air" and hydrogen-helium-"air" systems over a range of pressures. These data contributed toward a general consideration of the pressure dependence of flashback for several fuel-oxidant systems. No relation was found between the critical boundary-velocity gradient at 1 atmosphere and its pressure exponent. However, the critical flashback gradient at a pressure of 1 atmosphere and an equivalence ratio of 1 decreased exponentially with the reciprocal of the adiabatic flame temperature in the manner of a chemical reaction rate.

From author's summary

2814. Kurz, P. F., Behaviour of inverted propane-hydrogen sulphide flames at the blow-off limits, *Combustion and Flame* **1,** 3, 257–263, Sept. 1957.

Blow-off studies with inverted propane-hydrogen sulphide flames show that at the lean limit propane appears to contribute its proportionate share to the stability of the mixed fuel, whereas the burning of hydrogen sulphide is inhibited in the presence of propane. At rich blow-off, hydrogen sulphide appears to inhibit the burning of propane in all mixtures, and when the fuel mixture contains more than 50 volume per cent hydrogen sulphide the component fuels are mutually inhibiting.

From author's summary

2815. Spalding, D. B., Predicting the laminar flame speed in gases with temperature-explicit reaction rates, *Combustion and Flame* **1,** 3, 287–295, Sept. 1957.

Paper contains three graphs from which flame velocity estimates may be derived for chemical processes which can be described by a single effective over-all reaction step, e.g., one-step reactions, chain reactions obeying the steady-state approximation, chain-branching reactions with Lewis number of unity. Procedures are described also for introducing corrections when the Lewis numbers of reactants and products are not equal to unity.

S. S. Penner, USA

2816. Spalding, D. B., One-dimensional laminar flame theory for temperature-explicit reaction rates, *Combustion and Flame* **1,** 3, 296–307, Sept. 1957.

Exact numerical solutions of the laminar flame equations with Lewis number of unity are given for one-step over-all chemical reactions. The exact solutions are compared with approximate analytical results obtained by using the procedures of Wilde, Adams, von Kármán and Penner, Zeldovich and Frank-Kamenetsky, and Corner. The preceding list of authors corresponds, according to Spalding, to the order of decreasing accuracy of the approximation procedures used for solution of the flame equations.

Spalding's analysis is based on the assumption that the exponential Arrhenius reaction-rate function can be replaced by a reaction-rate law which gives zero conversion rate at the cold boundary. A careful comparison of approximate methods of solution [Th. von Kármán, "A comparison of approximation procedures for the solution of the flame equations," Pasadena, 1957] shows that Spalding's relative ratings do not apply at all to conventional Arrhenius reaction-rate functions.

Spalding's identification of analytical procedures is of doubtful significance. Thus Spalding's Eq. [62] should read $g = r^{n+1}$, although it is more correct to attribute $g = r^n$ to Adams' method in Eq. [65], as Spalding actually does. What Spalding calls Adams' method is practically the second Boys-Corner approximation and bears no relation to Adams' actual procedure when the error in Eq. [62] is not made; what Spalding calls Wilde's method bears no obvious relation to what this author does for a reaction rate with an Arrhenius factor (Wilde should use $g \sim r^{n+1}$); finally, the procedure to which Spalding refers as the Kármán-Penner method actually constitutes a first approximation in an iteration process which was developed for chemical reactions with large activation energy.

S. S. Penner, USA

2817. Lezberg, E. A., Preliminary investigation of propane combustion in a 3-inch-diameter duct at inlet-air temperatures of 1400° to 1600° F, *NACA TN* 4028, 8 pp. + 1 table + 10 figs., July 1957.

Ignition delays and combustion efficiencies were determined for propane injected into a heated airstream. Spontaneous-ignition delays of 0.007 to 0.049 sec occurred with a single-tube injector at temperatures between 1395 and 1585 F. These results followed the Arrhenius relation with an apparent activation energy of 43 kilocalories per mole.

Two types of flames were observed for multipoint fuel injectors: a diffusion flame that stabilized at the injector orifices, and an ignition-stabilized flame that formed a flame front downstream of the fuel injector. Combustion efficiencies for the diffusion flames increased with increasing fuel-air ratio to values of 90 to 100% at a fuel-air ratio of about 0.01, with burner lengths 6 inches or longer. Efficiency decreased markedly at burner lengths below 6 inches. Decreasing pressure had only a small effect on the efficiencies. Air temperature had no effect in the range investigated. Combustion efficiencies for the ignition-stabilized flames were below 70 to 80% for burner lengths of 18 inches or less and were strongly dependent on burner length and temperature. Efficiency decreased as burner pressure was lowered.

From author's summary

2818. Norrish, R. G. W., and Zeelenberg, A. P., The combustion of hydrogen sulphide studied by flash photolysis and kinetic spectroscopy, *Proc. roy. Soc. Lond.* (*A*) **240,** 1222, 293–303, June 1957.

The combustion of hydrogen sulphide has been investigated by the method of kinetic spectroscopy and flash photolysis. If no large excess of inert gas is present, the reaction produces sulphur dioxide. The reaction has been shown to take place in steps in which the radicals SH and OH participate. Simultaneously with the appearance of the sulphur dioxide, a light emission has been observed which is attributed to the process: $SO + O \rightarrow SO_2 + h\nu$. In the presence of a large excess of inert gas the reaction results in S_2O_2. It has been shown that the formation of S_2O_2 is favored by a low temperature.

Flash photolysis of hydrogen sulphide, sulphur dioxide, and S_2O_2 has also been investigated. In the last two cases the absorption spectra disappear temporarily. Inert gas prevents the disappearance of the SO_2, but not that of S_2O_2. From an analysis of results a mechanism for the combustion of hydrogen sulphide is derived and discussed.

From authors' summary

2819. Pearson, S. W., Thring, M. W., and Chesters, J. H., Combustion and heat transfer in an open-hearth furnace, Iron and Steel Inst. Spec. Rep. no. 59, 82 pp., 1956.

With an extensive research team the authors have gathered considerable quantitative data on the thermodynamic behavior of an open-hearth furnace during casting. Extensive data on flame emissivity and temperature, combustion and course of mixing, roof and metal temperatures, as well as heat losses and an over-all heat balance are presented. Each subject is treated in detail and experimentally measured data are supported by model method computations and theoretical considerations.

The task the authors have chosen is complicated by the myriad of factors involved and the logical critique, paraphrased from the text, is that certain conclusions relate to the particular furnace and conditions existing in it and are not of universal application. Justification of the project is that "the particular furnace studied being of an unusually simple design—single uptake, single burner, and virtually symmetrical furnace chamber—provides an excellent starting point for future calculations and development."

Certainly the data are qualitatively useful but, more important, the results contribute to an understanding of the combustion and heat transfer process. On this basis the report is recommended to those involved with the design or operation of open-hearth furnaces. The authors are to be commended on their well-organized approach and careful regard given to the many details involved in preparation and collection of the data.

J. W. Hlinka, USA

2820. Miele, A., Optimum burning program as related to aerodynamic heating for a missile traversing the earth's atmosphere, *Jet Propulsion* **27,** 12, 1231–1240, Dec. 1957.

A rocket engine with variable thrust drives a given vehicle along a path of prescribed geometry. What thrust program minimizes the

difference between final and initial values of a specified function G of the vehicle coordinates? Conventional treatments constrain minimization only by equations of motion; present one adds heat balance equation constraint. Assumptions: thrust along path, rocket equivalent exit velocity constant, all mass flows possible between zero and a specified maximum, aerodynamic lag disregarded, constant gravity, straight inclined trajectory (near-vertical), induced drag neglected; atmospheric properties specified, heat transfer by one-dimensional (inward) flow at each point of skin (implying conservative results). Dependent variables are mass, altitude, velocity, skin temperature and an engine parameter related to thrust level; independent variable is time. Function to be minimized can be quite general with specific examples of temperature, propellant consumption. For problems not involving time, explicit differential equation obtained for optimum mass flow which must be treated numerically. Making further restrictive assumptions, closed solutions are obtained for class of problems where $G = G(m,b,V,T)$. Several numerical examples are given to illustrate the approximate theory.

R. E. Roberson, USA

2821. Westenberg, A. A., Present status of information on transport properties applicable to combustion research, *Combustion and Flame* **1,** 3, 346–359, Sept. 1957.

A survey is presented of the available experimental and theoretical information on molecular diffusion and thermal conductivity coefficients likely to be of interest in low-speed combustion studies. Emphasis is placed on high-temperature data and practical methods of calculation. Reliability of present information is discussed, and the great lack of experimental data is pointed out. No pretense of a complete summary is made, but most of the best existing sources are indicated. Consideration of viscosity or dense gas effects is specifically excluded.

From author's summary

2822. Tucker, M., Interaction of a free flame front with a turbulence field, *NACA Rep.* 1277, 19 pp., 1956.

[See AMR **8,** Rev. 2905]

2823. Vulis, L., Some directions of development and problems of the heat theory of combustion (in Russian), *Vestnik Akad. Nauk Kaz. SSR* **5,** 89–101, 1953; *Ref. Zb Mekh.* no. 10, 1956, Rev. 6492.

A short examination of some problems of the heat theory of combustion.

The diffusion theory of combustion is taken as an example. The determining role of the gas dynamics of the flow for certain heating processes is noted.

M. A. Peshkin

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2824. Hubbard, E. H., The application of experimental methods developed in the Flame Research Organization, *Ingenieur* **69,** 37, 121–130, Sept. 1957.

Certain techniques and instruments such as suction pyrometers, pitot tubes and gas sampling methods used by the Foundation are described, and suggestions are made as to how they can be easily used or developed in industrial conditions.

From author's summary

2825. Van Langen, J. M., Practical heat technological research on industrial furnaces, *Ingenieur* **69,** 33, 109–114, Aug. 1957.

Experimental application of the results of flame research is described viz., the examination of the phenomena of heat transport and gas flow in the Siemens-Martin furnace. Four phases are to be distinguished: measurements in the Siemens-Martin furnace itself, in the experimental furnace, in a model of the experimental furnace, and in a model of the Siemens-Martin furnace. The aim was to establish laws and criteria of similarity in order to explain and to predict the behavior of the Siemens-Martin furnace by means of measurements in the latter model only. The progress in this endeavour is described.

From author's summary

2826. Gordon, A. S., Bennett, A. L., and Davidson, N., A photoelectric study of the light emission from carbon monoxide-oxygen thermal explosions, *Combustion and Flame* **1,** 1, 19–24, Mar. 1957.

The mobilities of ions at various points in a diffusion flame were observed by the sheath method in an attempt to discover in what part of a flame the ions were produced and how they combined with other molecules or radicals.

From authors' summary

2827. Moutet, A., Instantaneous measurement of flame temperature (in French), *ONERA Publ.* no. 88, 66 pp., 1957.

Author describes first the existing main optical methods for measuring flame temperatures with particular reference to the spectroscopic reversal line method and its latest improvements for the direct and instantaneous recording of temperature.

Short-time lag devices used for the study of combustion instabilities are also described. They all lead alternatively or continuously to the finding of the reversal point through the following three parameters: y_1 = energy of reference lamp at wave length y; y_2 = energy of reference lamp after passing through the flame; y_3 = energy of the flame only at same wave length. Typical recordings obtained for each device shown are given for various combustions. Physical meaning of temperatures measured is discussed in the case of heterogeneous flames.

From author's summary by R. Delbourgo, France

2828. Torda, T. P., Notes on problems in combustion instability of rocket motors, *Aero. Engng. Rev.* **16,** 11, 34–37, Nov. 1957.

Rather than a survey of the state of the art, this paper conveys some of the thinking of the author and describes briefly work by the Mechanical Engineering Department of the Polytechnic Institute of Brooklyn.

From author's summary

2829. Havemann, H. A., and Mahadevan, K., Experimental studies on a cyclone chamber for combustion and gasification of heavy liquid fuels, *J. Inst. Fuel* **30,** 192, 26–39, Jan. 1957.

Report presents an account of an experimental investigation into the operational behavior of a small cyclone chamber meant to utilize, as completely as possible, heavy liquid fuels. Design features are given for an air weight flow of ¼ lb per second. The experimental results obtained with different types of secondary-zone fuel injection systems for light diesel oil and preheated bunker oil are presented. The advisability of injecting the secondary-zone fuel at the periphery of the volute, with the direction of the gas flow, is established. Nearly complete combustion of fuel particles is achieved irrespective of the type of liquid fuel even when the air/fuel ratio is stoichiometric. Gasification is possible if the air/fuel ratio is gradually reduced to less than the particular stoichiometric ratio. The pressure-drop in the plant is comparatively low, being equal to seven or eight times the dynamic head of the gases at entrance to the volute.

From authors' summary

2830. Ghosh, B., Studies on combustion of pulverized coal: Part I. Physico-chemical aspects, *J. sci. indust. Res., India* **16,** 10, 464–469, Oct. 1957.

Following a discussion of the ignition of pulverized-coal flames after Nusselt and Traustel, the conditions and the heat balance of a stable flame are considered under certain simplified conditions. By solving this equation for the burner mouth velocity a two-term

expression is derived, being determined by the flow of heat due to conduction for the first, due to radiation for the second, term. A quantitative evaluation would only be possible if all factors involved are known. W. Gumz, Germany

2831. Ray, N. K., Basu, D., and Ghosh, B., Studies on combustion of pulverized coal: II. Combustion within a furnace, *J. sci. indust. Res., India* **16,** 10, 470–474, Oct. 1957.

Measurements of a small pulverized-coal flame (5-mm burner mouth diam) burning in an electrically heated furnace of 3-in. and 12-in. height indicate the importance of radiant heat transfer. Furnace walls had to be heated to 850C for Assam and Raniganj I coal (1.9–3.5% H_2O, 9.5–15.5% ash, 37.3–37.7% vol. matter), 900C for Raniganij II (28.5% vol. matter), and 950C for Jharia coal (21.5% ash, 19.7% vol. matter). Burner mouth velocities could be varied between 40 to 150 cm/sec. When a flame is established, furnace temperature can be lowered considerably. In reviewer's opinion, deviation from conditions of full size-furnaces is considerable. W. Gumz, Germany

Acoustics

(See also Revs. 2693, 2765)

2832. Wells, C. P., and Leitner, A., A Lebedev transform and the "baffle" problem, *Quart. appl. Math.* **15,** 4, 430–434, Jan. 1958.

The problem of sound radiated by a vibrating circular disk in an infinite, rigid baffle is solved by the application of a new transform [N. N. Lebedev, *Dokladi Akad. Nauk SSSR* **58,** p. 1007, 1947]. The radiation is represented in terms of spherical waves, and the complete eigenfunction expansion for the velocity potential is given. Earlier solutions of this problem are due to Sommerfeld (in terms of cylindrical waves) and to Bouwkamp (in terms of spheroidal waves). R. Heller, USA

2833. Olsen, H., Romberg, W., and Wergeland, H., Reaction of sound waves and its application for absolute measurement of intensity (in English), *Tekn. Skr.* no. 17, 12 pp., 1957.

Recent developments in ultrasonics have made valuable studies of acoustic radiation pressure possible, by measuring the thrust exerted by sound waves on suitable test bodies. Authors' have treated the case of a rigid sphere suspended as a pendulum in detail, deriving numerical values necessary for determination of absolute intensity of a sound wave. Reference is made to experiments at the Institute of Physics, University of Oslo, and previous research of L. V. King. The agreement with King's analysis is partly fortuitous and not valid for more general boundary conditions. Limiting cases of long and short wave-lengths are thoroughly discussed, and formulas for practical use are derived. Further references are cited to F. Borgnis who made a comprehensive survey for earlier work in this field, L. Brillouin, C. Eckart, P. Westerwelt who considers the viscous forces acting in addition to the radiation pressure, H. Wergeland, and to formulas of W. Magnus and F. Oberhettinger. J. J. Polivka, USA

2834. Kurbjun, M. C., Effects of blade plan form on free-space oscillating pressures near propellers at flight Mach numbers to 0.72, *NACA TN* 4068, 20 pp., Aug. 1957.

In order to obtain information on the effects of blade planform on the near-field noise of propellers, a series of measurements have been made of the oscillating pressures near a tapered-blade-planform propeller at flight Mach numbers up to 0.72. Flight Mach number, power, and engine speed were controlled in an attempt to duplicate the conditions of the flight investigation reported in *NACA TN* 3417, where oscillating pressures were measured near a rectangular-blade-planform propeller.

The tapered-blade propeller produced lower sound-pressure levels than the rectangular-blade propeller for the low blade-passage harmonics (frequencies where structural considerations are important) and produced higher sound-pressure levels for the higher blade-passage harmonics (frequencies where passenger comfort is important).

The effects of flight Mach number on the oscillating pressures produced by the tapered-blade propeller are the same as were found for the rectangular-blade propeller of *NACA TN* 3417. The lower blade-passage harmonics tend to decrease slowly with increase in flight Mach numbers up to approximately 0.5 and then to increase rapidly at higher Mach numbers. The sound-pressure levels of the higher harmonics of the propeller noise increase at a higher rate than do the lower harmonics with increase in flight number.

Relatively small changes in noise levels are observed for large changes in power settings at the design speed, flight Mach number of 0.5, of the propeller. This observation and the results of the calculations made in United Aircraft Corporation Research Department Report R-0896-2 indicate that effects other than blade loading noise, such as thickness noise, are producing noise levels of at least the same magnitude as the blade loading noise.

At speeds above a flight Mach number of 0.5, the propeller is operating above the design speed. The effect of this off-design condition in the present investigation would be to overemphasize the thickness noise relative to the blade loading noise that would normally be found in a propeller designed to operate at these higher speeds. From author's summary

2835. Kurbjun, M. C., Noise survey of a full-scale supersonic turbine-driven propeller under static conditions, *NACA TN* 4059, 8 pp. + 1 table + 7 figs., July 1957.

Over-all sound-pressure levels and frequency spectra of the noise emitted from a full-scale, 7.2-ft-diam 3500-rpm, three-blade, supersonic propeller mounted on a turbine-powered airplane have been obtained under static conditions at stations about the propeller at a 100-ft radius.

The results of this investigation are compared with the results of *NACA TN* 3422 [AMR **9,** Rev. 589] for a propeller of conventional design. The comparison shows that the high-rotational-speed propeller produced an over-all sound-pressure level of approximately 14 decibels more at the maximum-level station than the low-rotational-speed propeller. The spectrum of the noise of the high-rotational-speed propeller isgenerally flatter than the spectrum of the low-rotational-speed propeller, and the second, third, fourth, and fifth harmonics are higher than the first harmonic. The low-rotational-speed propeller displayed the maximum level in the first harmonic with a rapid drop in sound-pressure levels as the order of the harmonic increases.

Variations in power produced, in general, the variations in over-all sound-pressure levels predicted by theory. The effect of a power increase on the spectrum of the noise is to raise the levels of the lower harmonics. A small reduction in the over-all sound pressure was obtained by lowering the propeller tip Mach number from 1.2 to 0.99; the reduction was in agreement with the scale-model results of *NACA Rep.* 1079. Analysis shows the noise reduction was afforded by reductions in the noise levels of the harmonics above the third harmonic.

From author's summary

2836. Coles, W. D., and North, W. J., Screen-type noise reduction devices for ground running of turbojet engines, *NACA TN* 4033, 9 pp. + 12 figs., July 1957.

The previously reported [*NACA TN* 3452] advantages of screens placed across the jet as a means of suppressing jet noise during

ground running were somewhat offset by increased noise levels ahead of the engine. This disadvantage has been overcome by a combination screen and muffler which effectively eliminates these increases and gives substantial additional suppression throughout the sound field. Maximum sound pressure levels at 200 ft were reduced to 104 decibels (a 16-db reduction), and the over-all sound power was reduced by 12 decibels. Reductions of at least 4 decibels and as much as 17 decibels were obtained in the spectrum power levels.

Air-jet tests showed negligible reduction in sound generation with additional screens. Both air-jet and engine tests showed airfoil-vane jet diffusers to be less effective than screens.

The large reductions obtained by using screens show that noise generated inside the engine (nonafterburning) by the turbine or by combustion contribute only a minor part of the total noise.

From authors' summary

2837. Dauphinee, T. M., Acoustic air pump, *Rev. sci. Instrum.* **28,** 6, p. 452, June 1957.

An air-circulating pump that works on the principle of acoustic wind is described. The pump will move up to 700 l/min of air when powered at 60 cps by a 4-in. loudspeaker. The speaker diaphragm is the only moving part and could be protected by inert material when it is desired to seal in and circulate corrosive gases. Alternating current pickup from the speaker is so small as to be negligible in most instances.

From author's summary

Ballistics, Detonics (Explosions)

(See also Revs. 2444, 2487, 2820, 2838)

2838. Kapur, J. N., Unified theory of internal ballistics, *Quart. J. Mech. appl. Math.* **11,** 1, 98–111, Feb. 1958.

Author establishes a composite, double-chamber gun that incorporates recoilless and high-low features. Basic thermodynamic equations for state of propellant gas, gas flow, and projectile motion are developed. By suppressing various features of this model, the general equations are reduced to those for all other types of guns, including high-low recoilless, orthodox guns, and solid fuel rockets. Consideration is given to both before and after all-burnt conditions. Analyses are carried only to the extent of solving the principal problem of interior ballistics.

C. E. Balleisen, USA

2839. Seifert, H. S., The performance of a rocket with tapered exhaust velocity, *Jet Propulsion* **27,** 12, 1264–1266, Dec. 1957.

The effect of varying the exhaust gas velocity of a rocket so that the exhaust gases possess little or no kinetic energy in an earth observer's frame of reference is examined. Under certain boundary conditions an improvement in terminal velocity of 10 to 20% is obtained relative to a rocket with fixed exhaust velocity. Given a fixed budget of energy per unit payload mass, the optimum mass of working fluid into which to distribute this energy (for maximum burnout speed) is calculated as a function of gravitational field strength.

From author's summary

2840. Charters, A. C., Denardo, B. P., and Rossow, V. J., Development of a piston-compressor type light-gas gun for the launching of free-flight models at high velocity, *NACA TN* 4143, 95 pp., Nov. 1957.

Paper presents detailed information on the Ames Aeronautical Laboratory helium gun. In the initial firing tests described, a velocity of 15,400 fps was reached for a cylinder projectile weighing 0.2 grams, launched at 70,000 psi chamber pressure, and with the barrel evacuated till 8 mm Hg.

Reviewer also calls attention to the version developed by U. S. Naval Ordnance Laboratory, using helium plus reacting hydrogen-oxygen as propellant gas mixture. Employing a solid plug instead of a rupture disk to cap the barrel, the chamber is converted by hangfire into a high-pressure high-temperature autoclave. Next to the high pressure, peak temperatures as high as 2700 K may be obtained, conditions which will greatly arouse the interest of the chemical engineer [see: *Indust. Engng. Chem.* **49,** no. 12, Dec. 9, 1957].

J. D. Yap, Holland

2841. Clemow, B. J., Guided weapon engineering, *Aircr. Engng.* **24,** 343, 258–267, Sept. 1957.

2842. Schwartz, A. B., Malick, S., and Friesen, J. R., Measurement of the moment of inertia of missile-type bodies, *Aircr. Engng.* **24,** 343, 271–274, Sept. 1957.

2843. Turkovsky, V. A., The problem of the Brachystochrone in a resistant medium (in Russian), *Izv. Kievsk. politekhn. in-ta* **12,** 111–120, 1953; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8027.

The derivation is given of the parametric equations of the curve along which a ponderous material point will descend in the shortest time between two points, while moving in a resistant medium. The resisting force is assumed proportional to the square of the velocity of the point.

A mistake has been made in the projection of the equation of motion on the tangent to the trajectory.

V. V. Petkevich

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2844. Turkovsky, V. A., The Brachystochrone (short-period ballistic curve) in the presence of friction (in Russian), *Izv. Kievsk. politekhn. in-ta* **12,** 90–95, 1953; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8026.

The derivation is given of the parametric equations of an irregular curve along which a ponderous material point will fall from one position in the shortest time.

In the projection of the equation of motion on the tangent to the trajectory, a mistake has been made, leading to incorrect expression for the coefficients of the parametric equations of the required curve.

V. V. Petkevich

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2845. Besse, L., Course in external ballistic (in French), *Mem. Artill. fr.* **31,** 2, 391–446, 1957.

Soil Mechanics, Seepage

(See also Revs. 2568, 2569, 2638, 2661, 2872)

Book—2846. Bally, R. J., Electrical methods for improving soil properties in construction works (in Romanian), **Romania,** Editura Academiei Republicii Populare Romine, 1957, 246 pp.

Book discusses electrical methods for improving soil properties and their practical application to construction works. Author reviews the existing literature and presents the results of his own scientific research.

The first part describes certain general features of the problem, that is, a history of these electrical methods in general, and their practical application to construction in particular. Further a new classification of the methods for improving soil properties is presented. The first part concludes with a chapter devoted to the electrical resistivity of clayey soils, pointing out especially the influence of humidity upon their resistivity and the way in which

the process of applying an electrical potential through various soils develops.

The second part deals with the effects produced by the application of an electric potential through the soils. Special attention is given to the development of electro-osmotic processes when applied to compressible soils. Various aspects of electro-osmosis and its characteristic effects caused by the soil compressibility are underlined. One paragraph is intended as the mathematical interpretation of the process, an original method being provided in order to mathematically explain this phenomenon. Furthermore, the effect of an electrical potential upon the content of the adsorption complex of clayey soils and the cementing processes produced by applying an electric current through the soil are discussed. In another chapter, the change of soil properties under the action of a d c supply is examined both in a first stage in which the prevalent effect is caused by the electro-osmosis, and in a second one which corresponds to an intense development of electro-chemical transformation of the soil. The influence exercised by the soil content by the characteristics of the applied electric supply upon the effect of the electrical treatment of soils is discussed. In view of practically applying these methods, certain electro-osmotic coefficients are proposed which might characterize the soil behaviour when the electrical potential is applied.

The third part presents various practical applications of the improvement of soil properties for construction purposes. A distinction is made between the methods based upon the electro-osmotic process (electro-drainage, electrical compaction, change of ground water direction by means of electro-osmosis) and those which lead to an electrochemical consolidation of the soils (electro-chemical consolidation by means of aluminum electrodes, or various chemical additives). The method of the electro-silicification of soils is dealt with separately.

Details concerning the laboratory tests as well as the practical application of the electrical methods for improving soil properties made by various investigators are indicated in the appendices.

From preface

2847. Lundgren, H., Dimensional analysis in soil mechanics, *Acta Polyt.* no. 237, 32 pp., 1957.

Paper clearly shows how dimensional analysis may be employed in solving a variety of soil mechanics problems. The basic laws of similitude are applied to solve problems relating to consolidation, settlement, vibration testing, and pile driving, in clays and sands. The procedures and utility of using models to obtain reliable prototype information is also discussed. The only criticism—and a minor one indeed—is that the author does not emphasize, sufficiently, that the successful use of the process often demands a considerable amount of engineering judgment and physical insight in selecting which physical parameters should be used in the analysis. Nevertheless, author is to be commended in advocating the use of dimensional analysis in a field which, until quite recently, made little or no use of this valuable tool.

M. Lemcoe, USA

2848. Trollope, D. H., and Lee, I. K., The performance of a laboratory earth pressure cell, *Austral. J. appl. Sci.* **8**, 2, 84–97, June 1957.

Paper describes an experimental investigation of the performance of an earth pressure cell designed to record earth pressures within the range 0 to 1 lb in.$^{-1}$ under laboratory conditions.

The gages were calibrated under fluid (air) pressure and sand pressure. Under air calibration, a stable linear calibration characteristic was established. Under sand loading, however, the gages recorded a mean pressure corresponding to only 93% of the nominal height of sand over the gage. This under-registration was explained by a recently developed arching theory, and it is concluded that, for measurements of this type, a unique value of earth pressure cannot be recorded: the average value derived from several tests must be used.

From authors' summary

2849. Lee, I. K., and Brown, E. B., The design and construction of a laboratory earth pressure cell, *Austral. J. appl. Sci.* **8**, 2, 71–83, June 1957.

The present paper describes the development and construction of an electro-magnetic-type earth pressure cell suitable for pressure measurements along a flexible boundary.

From authors' summary

2850. Gribanov, I. I., Investigation of the stressed state of hydrotechnical installations of closed frame construction with reference to the elastic properties of the soil (in Russian), Avtoref. diss. kand. tekhn. nauk in-ta sooruzh. AN. UzSSR, Tashkent, 1956; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8580.

2851. Leese, G. W., Soil compaction investigation; effect of lift thickness and tire pressure, U. S. Army Wwys. Exp. Sta., Corps of Engrs., Vicksburg, Miss., Tech. Mem. 3–271, Rep. no. 8, 35 pp. + illus + tables, Oct. 1957.

Lean clay test fills were constructed using a rubber-tired roller with tire pressures of 90 and 150 psi and wheel loads of 25,000 and 31,250 lb, respectively, to study the effects of variation in lift thickness on the compaction of this material by a rubber-tired roller. It was found that the 90-psi roller can effectively compact loose lift thicknesses up to 14 in. at optimum water content, whereas the 150-psi roller can effectively compact loose lift thicknesses of only 9 in. or less at optimum water content because of the greater sinkage or rutting it produces in the loose soil. In determining the economy of compaction in thicker lifts, the fact that heavy rollers with high-pressure tires require extra tractive effort and possibly precompaction with lighter rollers to prevent high initial sinkage and shoving in the loose lift may outweigh the advantage of increased lift thickness.

From author's summary

2852. Vialov, S. S., and Tsytovich, N. A., Binding properties of frozen ground (in Russian), *Dokladi Akad. Nauk SSSR* (*N.S.*) **104**, 4, 527–529, Oct. 1955.

2853. Tsytovich, N. A., Rules for determining the cohesion forces in frozen soils (in Russian), Reports of the Laboratory for Research on Frozen Soils, no. 2, Moscow, Adad. Nauk SSSR, 1954, 162–175; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7819.

Author suggests that the cohesion of frozen soils can be measured by a ball indentation test. An instrument for the purpose is described and illustrated, with rules for its use for cohesion measurements, both in the laboratory and in the field.

P. L. Ivanov

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2854. Tsytovich, N. A., Some general problems in the methodology of physical and mechanical investigation of frozen soils (in Russian), Report of the Laboratory for Research on Frozen Soils, no. 2, Moscow, Akad. Nauk SSSR, 1954, 5–15; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7820.

A classification is given of the fundamental physical states of frozen soils, the terminology thereof is revised, and their physical and mechanical characteristics more closely defined.

P. L. Ivanov

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2855. Geertsma, J., The effect of fluid pressure decline on volumetric changes of porous rocks, *J. Petr. Technol.* **9**, 12, 331-340, Dec. 1957.

In order to obtain a better insight into the pressure-volume relationship of reservoir rocks a theory of pore and rock bulk volume variations is presented. The theory is independent of the shape of the pores but is restricted to isotropic porous media built up of continuous homogeneous matrix material. The main conclusion obtained from this theory is that only three elastic constants and three viscous constants are required for describing pore and rock bulk volume variations if the porosity is explicitly introduced into the treatment.

In addition, reasonable approximations are introduced for various types of reservoir rock, e.g., sandstones, limestones, and shales, which lead to further simplifications of the basic formulas. In consequence there is then a further reduction in the number of deformation constants which have to be determined experimentally. It is shown how measurements of these remaining deformation constants can be performed most conveniently.

Finally, the application of the theory to reservoir studies is discussed and the translation of experimental results obtained in the laboratory into reservoir behavior is considered.

From author's summary

2856. Mel'nikova, M. K., and Nerpin, S. V., A study of equilibrium conditions of moisture in dispersion systems in the presence of a gravitational field (in Russian), *Dokladi Akad. Nauk SSSR (N.S.)* **106**, 4, p. 615, Feb. 1956.

2857. Arkhangelsky, M. M., The calculation of pile-cluster foundations (grill foundations) for horizontal loading (in Russian), *Trudi Novosibire. in-ta inzh. zh.-d. transp.* no. 11, 102-129, 1955; *Ref. Zh. Mekh.* no. 10, 1956, Rev. 6959.

The general scheme of calculation of piled foundation grids is discussed, for the case where the above-ground structure can be regarded as a horizontal platform on elastically yielding supports, while the soil represents an elastic winkler foundation slab.

The case of a foundation slab on four vertical piles is analyzed in detail. Experimental results are given for the work of a single pile and a piled grill. A. M. Kochetkov

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2858. Verrigo, M. F., Calculation of the stresses in the ballast and on the foundation level of a permanent railway roadbed (in Russian), *Trudi vses. n.-i. in-ta zh.-d transp.* no. 97, 326-352, 1955; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7811.

The stresses in the ballast layer and the pressures on the formation level, transmitted through a series of sleepers resting on the surface, are determined. N. N. Ivanov

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2859. Gorbunov-Posadov, M. I., Schachter, O. Ya., Nad Koffman, V. A., Ground pressure on a rigid, sunk foundation and the free deformations of the foundation pit (in Russian), *Trudi n.-i. in-ta osnovaniy i fundamentov* no. 24, 39-80, 1954; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7680.

A solution is presented for the stress and strain distribution in an elastic semiplane with a rectangular cutout, loaded by its own weight and the following loading cases: (1) The pressure of a rigid die, filling the cutout and loaded by a centralized, vertical form; (2) the weight of a rigid slab, laid on the bottom of the cutout; (3) a vertically-acting load uniformly distributed over the bottom of the cutout.

The solution is arrived at by applying a series of fictitious loads along the perimeter of the cutout, distributed in such manner as to satisfy the conditions on the contour or boundary of a semiplane with a cutout. The determination of the stresses and strains due to these fictitious loads is based on the solution of the problem of a concentrated load acting near the boundary of an elastic semiplane. The distribution of the fictitious loads is approximately represented by algebraic series interrupted at terms of the fourth and lower orders. This leads to incomplete fulfilment of the boundary conditions, with deviations of the order of 10%. The analysis and the numerical examples presented enable an estimate of the accuracy of the solution, which appears to be adequate for practical purposes. K. A. Ksenofontov

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2860. Zhukovsky, E. Z., Problems of the calculation of flexible ferroconcrete foundations with allowance for the time factor (in Russian), *Nauch. raboty studentov Leningr. inzh.-stroit. in-ta* no. 2, 5-18, 1955; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7764.

The calculation of a flexible, ferroconcrete foundation resting on the ground is reduced to the calculation of an elastically tenacious slab on an elastically tenacious support [A. R. Rzhanitsin, "Some problems of the mechanics of time-dependent deformation systems," Moscow, Gostekhizdat, 1949]. An investigation is made of a simplified equation for the distorted axis of an elastic beam on an elastically tenacious support. The tenacity of the beam is considered by introducing a time-variable, elastic modulus into the solution. N. N. Shchetinin

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2861. Meschien, S. R., The creep problem in coherent soils (in Armenian), *Izv. Akad. Nauk ArmSSR, Ser. Fiz.-Mat., Estestv. i tekhn. Nauk,* **7**, 6, 51-59, 1954; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7822.

The results are presented of laboratory tests on the creep deformation in compression of a loam soil with varying initial humidity and constant initial porosity. The tests were made in a compression apparatus excluding lateral deformation of the sample. An expression is derived for the rate of creep, applicable in the testing of clay soils generally. P. L. Ivanov

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2862. Proceedings of the Second Conference on Building of the Hungarian Academy of Sciences (Foundation and Soil Mechanics Section) (in Hungarian), Magyar Tudomanyos Akademia Muszaki Tudomanyok, Osztalyanak Kozlemenyei, 19, 1/3, 1956.

The papers of Hungarian authors (see following reviews) and the discussions give an account of the development of soil mechanics and foundation engineering during the past ten years in Hungary.

At the beginning of the volume, Prof. Szechy gives a tribute to the late Prof. J. Jaky, who introduced this branch of knowledge to Hungary and was a leader and research worker of world-wide reputation in it. A. Kezdi, Hungary

2863. Biczok, J., Experiences gained with subsurface exploration methods (in Hungarian), 11-25.

Paper describes in detail experiments with sounding, geophysical methods—in this field, the reconnaissance of sound and weathered rock strata under the bottom of the Danube bed is of especial interest; undisturbed sampling; field use of radiactive isotropes; documentation of subsurface data.

A. Kezdi, Hungary

2864. Pogany, B., and Janik, J. A., Determination of moisture content of soils with scattered neutrons (in Hungarian), 27-36.

With the aid of a source of beryllium, through the reaction $[\gamma, n]$, authors determine the number of scattered neutrons in the soil and, using empirical calibration curve, the moisture content.

A. Kezdi, Hungary

2865. Kezdi, A., Results of theoretical research (in Hungarian), 71-84.

The introduction reviews the work of Professor Jáky in the field of earth pressure, strut loads, state of stress in perfectly plastic soils. The main part of the paper surveys the latest Hungarian results achieved in the theoretical field: earth pressure on shafts [Karafiáth]; on bridge abutements [Széchy]; slip surfaces in the plastic state of earth masses [Kopácsy]; plastic deformations in cohesionless soils [Kézdi]; stress-distribution problems [Kézdi]; application of the theory of consolidation to practical problems [Karafiáth, Rétháti]; effect of rate of loading on bearing capacity [Karafiáth]; theory of elasticity of soil-water system [Kézdi]; bearing capacity of piles and pile groups [Kézdi]; model laws and state of stress in unconfined and confined compression [Balla].

From author's summary

2866. Bazant, Z., Foundation problems of hydroelectric powerplants in the valley of the Vah (in Hungarian), 85-87.

Paper discusses methods used in strutting and dewatering of excavations; settlements.

A. Kezdi, Hungary

2867. Domjan, J., Landslides and earthworks (in Hungarian), 89-105.

Subjects included in this paper are stability of steep banks along the Danube, of the shores of Balaton, slides of cuts, in clay soils with mosaic structures; debris slides; slides caused by pore-water pressure.

A. Kezdi, Hungary

2868. Jaray, J., Problems of soil mechanics in road construction (in Hungarian), 107-141.

Author discusses: Field experiences; failures of pavements, causes and remedies. Compacting earth embankments. Methods used in pavement design. Stabilization of soils. Effects of specific surface of soil grains in stabilization; variations in the value of specific surface caused by chemical treatment.

A. Kezdi, Hungary

2869. Szechy, K., Experiences gained in the field of soil mechanics and foundation engineering by building structures (in Hungarian), 143-174.

Case histories are presented and evaluated in the following fields: strutting and dewatering of excavations; drainage with filter wells [multiple-stage setup] and with vacuum method; foundation with exchange of base layer of low bearing value with sandy gravel; settlements of buildings with spread foundations; piled structures; sinking of shafts using thyxotropic materials; freezing method; big floating caissons and open-box caissons, pneumatic foundations. Experiences in the construction of the underground of Budapest are described.

A. Kezdi, Hungary

2870. Tsitovich, N. A., Problems of soil mechanics in the field of foundations (in Hungarian), 51-70.

Paper presents new methods and theories used in the Soviet Union: Bearing capacity of soils, in the case of oblique loading [Sokolowski]; quick method for determining cohesion with pressing a steel ball into the soil [Tsitovich]; earth pressure on retaining walls [Sokolowski]; computation of settlements using an equivalent layer-thickness [Tsitovich]; primary and secondary consolidation of clays [Florin]; allowable settlements of buildings [Building Code of the Soviet-Union]; subsidence of losses [Abelev, Tsitovich]; behavior of rigid clays [Rosa].

A. Kezdi, Hungary

Micromeritics

(See also Revs. 2658, 2671, 2687, 2800, 2804, 2855)

2871. Dahl, N. J., On water supply from wells, *Acta Polyt.* no. 236, 17 pp. + 2 figs., 1957.

An interesting rigorous treatment of well theory takes into account percolation due to rain. Several worked-out examples make the monograph very useful.

J. M. DallaValle, USA

2872. Ruchkin, V. P., Computation of the bottom of reservoirs on an elastic monolayer basis (according to the theory of Professor V. Z. Vlasov) (in Russian), *Avtoref. diss. kand. tekhn. Nauk Mosk. inzh.-stroit. in-ta,* Moscow, 1956; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8483.

Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

2873. Seifulina, B. A., Numerical solution of certain problems on the control of the motion of a petroleum-bearing contour (in Russian), *Izv. Kazansk. fil. Akad. Nauk SSSR, Ser. fiz.-matem. i tekhn. Nauk* no. 8, 83-99, 1955; *Ref. Zh. Mekh.* no. 10, 1956, Rev. 6760.

Investigation of a case in which a homogeneous horizontal bed of constant force is bored by wells, the position of which is known, the viscosities of petroleum and water are considered identical, and the conditions of the bed are watertight. Initial G_0 and final G_s of the position of the petroleum-bearing contour are also known, each of the contours being given in accordance with the M points

$$(r_{0j}, \theta_j) \quad \text{and} \quad (r_{1j}, \theta_j) \quad (j = 1, 2, \ldots, M)$$

The law of contraction of the petroleum-bearing contour is given according to the rays $\theta = \theta_j$ in the form of equations

$$r_j^2 = r_{0j}^2 - (t/T)(r_{0j}^2 - r_{1j}^2) \quad (j = 1, 2, \ldots, M)$$

where T is the treatment time.

In the work of G. S. Salekhov [*Ref. Zh. Mekh.* 1956, Rev. 6756], an approximate solution of this problem is given with the aid of the method of minimum error. Author examines particular cases, especially: (1) With concentric contraction of a circular petroleum-bearing contour, the solution obtained being compared with the case when G_0 is given as continuous, (2) when G_0 and G_1 are symmetrical relative to the coordinate axes (i.e. are oval in shape), and the location of the wells is also symmetrical relative to both coordinate axes. Several numerical examples are given.

V. A. Karpychev
Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

2874. Salekhov, G. S., Method of solving the problem of simultaneous control of the motion of several petroleum-bearing contours (in Russian), *Izv. Kazansk. fil. Akad. Nauk SSSR, Ser. fiz.-matem i tekhn. Nauk* no. 8, 100-114, 1955; *Ref. Zh. Mekh.* no. 10, 1956, Rev. 6761.

It is shown that the method of minimum error used by the author in a similar investigation [*Ref. Zh. Mekh.* 1956, Rev. 6756] may be applied to the solution of the simultaneous control of the motion of several petroleum-bearing contours. In this case, as a standard

for the mean quadratic error, it is suggested that the sum of the separate errors for each petroleum-bearing contour be taken.

The use of the method is illustrated by an example.

V. P. Pilatovskii

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2875. Barenblatt, G. I., Some approximation methods for the theory of the unidimensional unsteady filtration of a liquid in the presence of elasticity (in Russian), *Izv. Akad. Nauk SSSR, Otd. tekh. Nauk* no. 9, 35-49, 1954; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7623.

An approximate method is discussed for the solution of unidimensional unsteady problems in the theory of heat transmission, which closely resembles the Karman method in the theory of the boundary layer. The method is most effective in cases where the relationship of the required function to the space coordinate x_1 is monotonous.

In the presence of elasticity, the filtration pressure P satisfies the equation of heat transmission. In this regard, the boundary conditions present the greatest interest, i.e.,

$$P(x, t)\big|_{x=L} = \varphi(t); \ \partial P/\partial x\big|_{x=L} = \psi(t)$$

For a linear flow, the required function P is represented by a polynomial in whole nonnegative powers of x, and with coefficients depending on the time t. For a plane-radial flow, the equation of P also contains a logarithmic term, while for a centrally symmetrical flow, a term of the form $x^{-1}f(t)$ is included. The coefficients are determined by utilizing the conditions at the boundaries, furnishing a system of algebraic equations and instantaneous integral relationships, resembling the relationships developed by L. G. Loitsianski [*Prikl. Mat. Mekh.* **13**, no. 5, 1949]. The latter further form a system of ordinary differential equations of the first order. At $t = 0$, it is assumed that $P = \text{const}$, or a relationship of $P(x)$ corresponding to a steady flow.

The (calculation) process is conventionally divided into two stages. It is assumed that, initially, the perturbation originating at the boundary is manifested only in a particular region $x < 1$, adjacent to the boundary. The values of 1 for $t = \text{const}$ grow with approach to the rigorous solution, i.e., with increasing number of terms in the equation of P. Selecting a particular number of terms for $P(x, t)$, the extent of the region $l(t)$ is determined from the aforesaid system of equations and conditions $\partial^m P/\partial x^m$, for $m > 1$ at the variable boundary $x = l(t)$.

The approach of the perturbation to the further (outer) boundary initiates the second stage, in which the unperturbed region vanishes.

The question of the optimum selection of the number and character of the boundary conditions, and the corresponding number and form of the integral relationships, is insufficiently explained.

The examples included, for the calculation of the inflow to a borehole, show the high efficacy of the method. A sufficient degree of accuracy was obtained when the pressure was represented in the form of a trinomial, satisfying the values of P and the first derivatives at the boundary, as well as the integral relationship for the material balance. D. A. Éfros

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2876. Gugnyayev, Ya. E., Laboratory experiments on the dynamics of formation of sand slopes (in Russian), *Trudi n.-i. in-ta osnovanyi i fundamentov* no. 25, 102-121, 1955; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8304.

Experiments are described, made in two launders ($10 \times 0.75 \times 0.6$, and $4 \times 0.5 \times 0.4$ m), on the reforming of slopes with a gradient of 1 : 3, 1 : 5, and 1 : 10, of seven loose soils with a particle size between 0.15 and 10 mm under the action of waves of a height of 3-12 cm, length 46-224 cm and a slope of 7-110, as well as a period of 0.55-1.60 sec. The following conclusions have been obtained.

Short waves (steepness under 40) erode the bank, while long waves (steepness over 40) build it up. If the profile of the slope has become relatively stable after prolonged action of short waves, typical furrows are formed: the depth of water over the furrow is 1.6, and over the accompanying ridge 0.75, of the height of the wave. The distance between the wave peak and the center of the furrow is equal to 3, and the width of the erosion zone of the slope is six wave heights.

In the protection of shores and earthworks from wave action in the presence of a constant water level, it is recommended to use a curvilinear or broken profile, such as obtained in the experiments described above.

Special experiments have shown that a protection by slabs laid on such a slope has been found more stable than a similar reinforcement laid on a rectilinear slope. B. A. Pyshkin

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2877. Valikanov, M. A., Basis for the gravitational theory of the movement of deposits (in Russian), *Izv. Akad. Nauk SSSR, Ser. geofiz.* no. 4, 349-359, 1954; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8393.

A brief exposition is given of the physical bases and the basic relationship of the conception developed by the author on the entrainment of suspended sediments by means of a turbulent stream, and also a comparison of the deductions from this theory with the experimental results. In the article by A. N. Kolmogorov it is shown that the basic energetical equation (10) of the study referred to appears to be inaccurate. G. I. Barenblatt

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

Geophysics, Meteorology, Oceanography

(See also Revs. 2478, 2568, 2718, 2720, 2721, 2746, 2751, 2901)

2878. Rasskazovsky, V. T., The problem of determining seismic loads of rigid structures (in Russian), *Izv. Akad. Nauk UzSSR* no. 5, 73-76, 1955; *Ref. Zh. Mekh.* no. 11, 1956, Rev. 7907.

The action of seismic shock on structures is analyzed on a model represented by a bar of constant cross section attached at its lower end and subjected only to deformation by shearing forces. It is demonstrated that the stress distribution under seismic shock differs substantially from that according to the so-called static theory of seismic resistance.

A. G. Nazarov

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2879. Wiegel, R. L., Beebe, K. E., and Moon, J., Ocean wave forces on circular cylindrical piles, *Proc. Amer. Soc. civ. Engrs.* **83**, HY 2 (*J. Hydraul. Div.*), Pap. 1199, 32 pp., Apr. 1957.

2880. Tolefson, H. B., Atmospheric temperature observations to 100,000 feet for several climatological regions of the northern hemisphere, *NACA TN* 4169, 7 pp. + 1 table + 1 fig., Nov. 1957.

2881. Stommel, H., and Veronis, G., Steady convection in a horizontal layer of fluid heated uniformly from above and cooled non-uniformly from below, *Tellus* **9**, 3, 401-407, Aug. 1957.

Pertubation solutions of the steady-state equations of heating and motion are derived for the three cases of no rotation, uniform

rotation, and rotation varying in a way similar to that of the Coriolis parameter with latitude. Friction and vertical advection of heat are included, but horizontal advection is neglected. The depth of the convection cells is determined by different factors in the three cases: viscosity, conductivity, horizontal scale, and initial stability in the first; mainly stability and horizontal scale in the second; and a combination of all parameters except friction in the third. In the latter case the cells tilt toward the west. Possible applications to trade wind and oceanic problems are suggested.

Reviewer finds the results interesting and mathematically satisfying but would like to have seen an evaluation of the size of the neglected horizontal advection term.

R. P. Pearce, Scotland

2882. Wooding, R. A., Steady-state free thermal convection of liquid in a saturated permeable medium, *J. Fluid Mech.* **2,** 3, 273–285, May 1957.

Paper states the partial differential equations which describe steady flow of fluid in saturated homogeneous permeable solid material under nonisothermal conditions. From these are derived the equations for flow of liquid (in particular, water), using suitable approximations and making use of empirical laws when necessary.

A numerical example of the method, with boundary conditions based on a geothermal area at Wairakei, New Zealand, is given. The results show features which are in fair agreement with temperature measurements made in the area.

From author's summary by D. M. Vestal, Jr., USA

2883. Buell, C. E., Synoptic scale motion of the atmosphere as two dimensional isotropic turbulence, *J. Meteor.* **14,** 5, 471–472 (Shorter Contributions), Oct. 1957.

2884. Phillips, O. M., On the generation of waves by turbulent wind, *J. Fluid Mech.* **2,** 5, 417–445, July 1957.

A new theory of the generation of waves by wind is presented. The classical theories of Kelvin-Helmholtz (shearing instability) and Jeffreys (sheltering) and more recent theories by Sverdrup and Munk (tangential stresses) and Eckart (normal pressure fluctuations) all fail to agree with observations in important respects. Kelvin's minimum velocity for instability growth is too high, the observed shelter coefficient too low by an order of magnitude, and tangential stresses give reasonable results only if current production is neglected. Eckart's theory fails to produce waves as high as those observed.

Phillip's theory is concerned with the resonance effect of the pressure fluctuations in a turbulent air stream on an initially calm water surface. These fluctuations are associated with eddies which grow and decay at varying rates, and can be defined mathematically by a spectral function, the Fourier-Stieltjes transform of the two-dimensional surface pressure covariance. The largest eddies are carried at the velocity of the main stream and the smaller ones at the lower velocities near the surface. A solution of the equations of motion is derived which gives the development in time of the transform of the surface displacement in terms of the transform of the pressure fluctuations. The form of this equation and its physical interpretation suggest consideration of the wave development in two stages, in which the time elapsed is respectively less or greater than the development time of the fluctuations. It is shown that in the initial stage the most prominent waves are ripples of wave length 1.7 cm corresponding to the minimum phase velocity $c = (4gT/\rho)^{\frac{1}{4}}$ (T the surface tension) moving in directions $\cos^{-1}(c/U)$ to that of the mean wind, U being the convection speed of the pressure fluctuations of wave length 1.7 cms (wind speed at height 1.7 cms approx.). The two directions of propagation would give the frequently observed rhombic pattern of ripples. The main stage of growth, studied by considering the asymptotic form (as t becomes large), of the solution of the equations of motion, is regarded as valid until the neglected nonlinear effects become important. An expression for the mean square surface displacement is derived showing it to be linear in time and proportional to the mean square pressure fluctuation. The main assumption is that the statistical properties of the wind are unaffected by the waves produced. Also viscosity is neglected. Agreement with available observations is good, and author suggests that the pressure resonance mechanism proposed in the theory may be the principal means by which energy is transferred from the wind to the waves.

To the reviewer this paper seems to rate in importance as high as any yet published on the subject. A new, fundamental and most encouraging avenue of approach has been opened up which must give new stimulus to both theoretical and experimental workers in this field.

R. P. Pearce, Scotland

2885. Westfall, J. R., Milwitzky, B., Silsby, N. S., and Dreher, R. C., A summary of ground-loads statistics, *NACA TN* 4008, 10 pp. + 10 figs., May 1957.

2886. Sakagami, J., On the atmospheric diffusion of gas or aerosol near the ground, *Ochanomizu Univ., Tokyo, natur. Sci. Rep.* **7,** 1, 25–61, 1956.

Author investigated the diffusion of gas or aerosol from various types of sources in spaces ranging from several meters to several kilometers horizontally, and within five meters from the ground vertically. His numerical data were burned at the end of World War II, but his theoretical analysis is presented here as of possible usefulness for agricultural, meteorological, and industrial sanitary applications. Theory for a differential equation is set up assuming that horizontal diffusion is according to vorticity transport, the vertical diffusion according to momentum transport theory, and that the coordinate system moves along with mean wind velocity. On this basis differential equations for diffusion are derived in detail. The experimental method also took the time change of diffusion into consideration. Apparatus comprised aspirators, speed-regulating device, rotary 12-way cock for taking samples, concentration recorder, and the necessary electric circuits. Procedure for analyzing the results is outlined. Some conclusions are made for a continuous line source of volatile liquid (burst pipe), and for a continuous three-dimensional source (concentrated explosion of a gas vessel, and of several vessels in an area).

K. J. DeJuhasz, Germany

2887. Turkovsky, V. A., The theory of the Brachystochrone in the presence of friction in a resistant medium (in Russian), *Izv. Kievsk. politekhn. in-ta* **12,** 131–129, 1953; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8028.

The contents of two articles by P. Vassilenko, published in 1934 and dealing with the problem of the brachystochrone in the presence of friction and resistance, are recapitulated. The erroneous character of the results arrived at by Vassilenko is demonstrated.

The errors made by Vassilenko can be disclosed more simply than done by this author.

V. V. Petkevich

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2888. Lineykin, P. S., On wind currents and the baroclinic layer over the sea (in Russian), *Trudi Gos. Okeanogr. in-ta* no. 29, 34–64, 1955; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8299.

The problem is examined of determining the wind currents and density distribution in a baroclinic channel of infinite depth. The

starting equations are:

$$\rho \frac{du}{dt} - 2\omega_z \rho v = u_x \frac{\partial^2 u}{\partial x^2} + \mu_z \frac{\partial^2 u}{\partial z^2} - \frac{\partial p}{\partial x}$$

$$\rho \frac{dv}{dt} + 2\omega_z \rho u = \mu_x \frac{\partial^2 v}{\partial x^2} + \mu_z \frac{\partial^2 v}{\partial z^2} \qquad [1]$$

$$0 = g\rho - \frac{\partial p}{\partial z}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} + \frac{\partial \rho w}{\partial z} = 0 \qquad [2]$$

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + w \frac{\partial \rho}{\partial z} = k_x \frac{\partial^2 \rho}{\partial x^2} + k_z \frac{\partial^2 \rho}{\partial z^2} \qquad [3]$$

wherein u, v, w = velocity components, ρ density, p pressure, μ_x, μ_z corresponding coefficients of horizontal and vertical mixing, k_x, k_z analogous coefficients of the diffusion density, ω_x, ω_z components of the angular velocity of rotation of the earth.

It is considered that the flow commences from a state of rest. Furthermore, for $t = 0$, the density is considered to be a linear function of the depth.

The boundary conditions for $z = \zeta$ are as follows:

$$\mu_z \frac{\partial u}{\partial z} = -T_x; \quad \mu_z \frac{\partial v}{\partial z} = -T_y; \quad p = p_0; \quad \frac{d\rho}{dz} = \rho_0 b = \text{const}$$

$$\frac{\partial \zeta}{\partial t} + u \frac{\partial \zeta}{\partial x} = w \big|_{z=\zeta}$$

For $z \to \infty$ all perturbations fade, and adhesion conditions are satisfied at the walls of the channel. The wind field and all hydrodynamic characteristics are regarded as being independent of the coordinate y. The sequence of solution of the problem is as follows: equations [2] and [3] are linearized; the pressure is eliminated by applying the hydrostatic equation; the resulting four equations are written in nondimensional coordinates; u and v are sought by expansion in sinusoidal series, and the remaining variables by the cosines of the variable x, after which the coefficients obtained are subjected to operational transformations relative to the variable t; and, finally, the representations obtained are regarded as an exponential function of the variable z. Exceedingly cumbersome expressions are obtained for the value sought. Author only examines in detail the case of $p = 0$; i.e., in fact only conclusively solves the steady-state problem, which does not require operational transformation.

A concrete calculation example is presented. The resulting polar diagram of the velocities perceptibly differs from the Ekman spiral in that, first, the velocities are distributed over an appreciably greater depth than Ekman's frictional depth; second, the angle of deflection of the current from the wind direction is rather smaller than Ekman's.

Author is of the opinion that in the upper part of the frictional layer the gradient-convection part of the current is somewhat less than that purely due to drift, but the latter fades in depth considerably faster. The depth of the baroclinic layer over the sea decreases with the mean velocity value in height. The paper includes other new conclusions. S. V. Zhak

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

Lubrication; Bearings; Wear

(See also Revs. 2617, 2685)

Book—2889. Barwell, F. T., Lubrication of bearings, London, Butterworths Scientific Publications, 1956, xii + 292.

The two-fold purpose of this interesting book, as stated by the author, is to review the most significant research in lubrication in the past fifty years, presenting the results in a form useful to both the research worker and the design engineer, and to impart to each of these individuals an understanding of the other's problems and points of view. It begins appropriately with a discussion of methods of characterization of surfaces. Succeeding introductory chapters deal with modes of lubrication, wear and its mechanisms, and viscosity. The main subject of the book is then introduced by an exposition of the principles of hydrodynamic lubrication as developed by Reynolds and Sommerfeld. The details of the latter's integration of the Reynolds equation are given in the appendix. There follows a short chapter on nondimensional methods and duty parameters. Recent experimental work on journal bearings, including oil flow and thermal considerations, are then presented. The application of the hydrodynamic theory and experimental results to the design of partial bearings and to general design principles follows. The next four chapters deal with journal bearings subject to variable loading, thrust bearings, concentrated contact bearings, externally pressurized bearings, and reciprocating sliders and oscillating bearings. The concluding chapters discuss bearing materials, lubricants for particular applications, and bearing test rigs.

In the opinion of the reviewer the author succeeds in accomplishing his stated purposes. First, 199 pertinent literature references are cited. Second, throughout his discussions of hydrodynamic lubrication, which is the principal subject, he maintains a basically scientific point of view and provides adequate mathematical development for all the results given. Thereby he provides a common language for the research worker and the bearing designer and achieves his aim of making each aware of the others' problems. By starting from first principles in his development of hydrodynamic theory he emphasizes the fundamental physical nature of work in this field.

Author directs the attention of the reader to important topics that are frequently dealt with empirically yet to which hydrodynamic theory can be usefully applied. Illustrative of these are the interaction of hydrodynamic and elastic (Hertz) pressures in line contacts and the theory of rheodynamic lubrication. The former will interest those working in gear and rolling contact bearing lubrication. The latter is an extension of the hydrodynamic theory in which the lubricant has a yield stress, and is applicable to grease lubrication under conditions where grease can be considered a homogeneous material.

The book is highly readable and all symbols are carefully defined. It is recommended to research workers and design engineers alike as a practical and scientific treatise on the lubrication of bearings. J. Givens, USA

2890. Pinkus, O., Solution of the tapered-land sector thrust bearing, ASME Ann. Meet., New York, N. Y., Dec. 1957. Pap. 57-A-152, 7 pp.

Reynolds equation in polar coordinates is solved on digital computer for sector thrust bearing with side leakage, assuming oil film shape of uniform taper in the circumferential direction, which implies slightly warped bearing surface. Minimum film thickness, oil flows, power loss, center of pressure are given in dimensionless form as function of dimensionless operating parameter. Performance calculations are discussed and a comparison made between these new results and results already published.

C. F. Kettleborough, New Zealand

2891. Sternlicht, B., Influence of pressure and temperature on oil viscosity in thrust bearings, ASME Ann. Meet., New York, N. Y., Dec. 1957. Pap. 57-A-275, 9 pp.

A theoretical study to determine the influence of pressure and temperature on oil viscosity in finite-length thrust bearings, this

paper points out that it is theoretically possible to determine the influence that pressure and temperature play on oil viscosity and, in turn, on bearing performance, by solving the energy and Reynolds equations simultaneously.

From author's summary

2892. Reinberg, E. S., Approximate stress calculation for the body of a segmental thrust bearing (in Russian), *Trudi Leningr. Korablestroit. in-ta* no. 17, 133–138, 1955; *Ref. Zb. Mekb.* no. 11, 1956, Rev. 7935.

The thrust bearing is treated as a bracket (cantilever) element loaded on the face end by a concentrated force, the normal stresses being calculated with allowance for shearing by transverse thrust.

B. N. Lopovok

Courtesy Referativnyi Zburnal, USSR

Translation, courtesy Ministry of Supply, England

2893. Emmerich, C. L., Measuring the preload of ball or roller bearings, ASME Ann. Meet., New York, N. Y., Dec. 1957. Pap. 57-A-43, 3 pp.

The preload actually attained in a duplex-bearing assembly may differ considerably from the design value because of minor changes in clearances, temperature, alignment, and so forth. Two methods are described for measuring the preload of a completed bearing assembly. From author's summary

2894. Needs, S. J., Viscosity-pressure effect on friction and temperature in a journal bearing, ASME Ann. Meet., New York, N. Y., Dec. 1957. Pap. 57-A-66, 4 pp.

The recently published report on investigations conducted by the ASME Research Committee on Lubrication on the increase in viscosity of lubricants with pressure brings up the question of the importance of this phenomenon in actual machine elements. The present tests were undertaken to show differences in friction and operating temperature when heavily loaded partial journal bearings were operated with two different oils in the same viscosity range but of widely different viscosity index and viscosity-pressure characteristics. It is hoped that the results obtained in this short study will lead to more comprehensive investigations, not only in the field of journal bearings but in other machine elements as well.

From author's summary

2895. Hartung, H. A., The pressure-viscosity effect: background, ASME Ann. Meet., New York, N. Y., Dec. 1957. Pap. 57–A–277, 4 pp.

Indications are that the pressure-viscosity effect is an important factor in certain cases of lubrication, notably heavily loaded elements. How important it is in the lubrication of lightly loaded elements we do not know, nor do we know what refinements in design may be possible by taking this factor into account. The three papers which follow this introduction are modern contributions to our knowledge in this area. Following them, the Research Proposal on the Machine Element Test program will be discussed. Hopefully, the result of this presentation and discussion will be a better understanding of the importance of this problem and the potential value to industry of a solution to it.

From author's summary

2896. Hull, E. H., Oil-whip resonance, ASME Ann. Meet., New York, N. Y., Dec. 1957. Pap. 57–A–169, 5 pp.

A series of oil-whip resonances has been demonstrated by applying a rotating force to a shaft running in a hydrodynamically lubricated bearing. These resonances appear at the ratios of shaft speed to exciting-force frequency of approximately 2, which is normal oil whip, 3, 4, 6, 8 .... and –2, –4, –6 ... The locus of the shaft center at these resonances traces a trochoid containing a number of loops related to the ratio of shaft speed and force frequency. Inside loops are formed at positive frequency ratios, outside loops at negative ratios. A qualitative explanation of the formation of these resonance figures is offered.

From author's summary

2897. Fisher, F. G., Allen, R. K., and Luvisi, G. W., Advancing adhesion—development and test of a chemical formulation to condition the running surface of rail for improved wheel-to-rail adhesion, ASME Ann. Meet., New York, N. Y., Dec. 1957. Pap. 57-A-130, 13 pp.

Chemical conditioning of rail running surface to overcome the effects of oil films is capable of producing a definite increase in locomotive adhesion. Extensive tests under operating conditions for more than a year have demonstrated its practicability and economy. Characteristics necessary to permit a chemical formulation to qualify for commercial use have been determined, and a practical method of application developed. The ultimate objective of application from the locomotive, as is done with sand, is being sought. From authors' summary

2898. Hersey, M. D., and Staples, C. W., Experiments on imperfect lubrication, ASME Ann. Meet., New York, N. Y., Dec. 1957. Pap. 57-A-62, 4 pp.

This brief paper is based upon a full report of certain experiments conducted for the ASME Research Committee on Lubrication. The experimental work comprised friction measurements under imperfect lubrication conditions up to loads of 10,000 psi or more. Eight lubricants were compared whose viscosity-pressure characteristics were known. A Kingsbury oil-testing machine was used, with bearing surfaces designed to eliminate hydrodynamic action so far as possible.

A condition of mixed lubrication prevailed, in which rapid wear of brass on steel occurred simultaneously with hydrodynamic effects. All tests were run well to the left of the familiar minimum point on the coefficient-of-friction diagram. Three methods of plotting the data were tried: (a) The direct plot of friction per unit area against load per unit area, holding the speed and viscosity constant; (b) a conventional plot of the coefficient against Z_1N/P, where Z_1 is the viscosity at atmospheric pressure, N the speed, and P the load per unit of projected area; and (c) a modified diagram in which each of the foregoing curves is replaced by a family of curves with different values of the pressure-viscosity parameter. This parameter, the product of load per unit area by the pressure coefficient of viscosity, helps to explain earlier discrepancies and to set up a better correlation.

The usual contrast between friction of fatty and mineral oils was confirmed, and it was concluded that boundary lubrication and wear are accompanied by hydrodynamic action on a microscopic scale. From authors' summary

Marine Engineering Problems

(See also Revs. 2469, 2505, 2574, 2755)

2899. Schuster, S., and Walinski, E. A., Contribution to the analysis of the propeller forces field (in German), *Schiffstechnik* **4**, 23, 195–204, Sept. 1957.

Author starts from measurements of the inequality (axial, circumferential and radial) of the relative velocities behind ship models, in the plane of the removed propellers. An analytical study of such results is made by means of Fourier series. A correction takes account of the propellers' presence.

The forces and moments acting on the propellers may then be theoretically deduced. For simplicity, author only computes the propellers' thrust.

The interest of such studies lies in the possibility of better efficiency of ship propellers, as well as in the choice of better forms of hull rears, and obtaining the minimum of hull vibrations.

R. G. A. Spronck, Belgium

2900. Maruo, H., The force of water waves upon a fixed obstacle, *Bull. Fac. Engng., Yokohama nat. Univ.* **5,** 11–31, Mar. 1956.

Author re-examines, by means of the momentum equation, problems of steady mean forces treated by Havelock [*Proc. roy. Soc. Lond.* **175,** 1940].

Reducing the computations by means of Green's theorem to surface integrals at a large distance from the obstacle he deduces general expressions for the case of vertical cylinders and applies them on the cases treated by Havelock.

In an approximation of a ship form, by means of sources replacing the obstacle, he discusses qualitatively the conclusions of Havelock as to the influence of the beam on the average wave force.

Finally, the steady mean force on vertical elliptic cylinders is treated in detail.

Reviewer does not believe that author has attained an essential simplification in the compution procedures.

H. J. Schoemaker, Holland

2901. Ivanov, A. A., The variability of wind waves in seas and oceans (in Russian), *Izv. Akad. Nauk SSSR Ser. geofiz.* no. 6, 557–560, 1955; *Ref. Zb. Mekb.* no. 12, 1956, Rev. 8300.

The inapplicability of statistical methods founded on Pearson's distribution curves to the analysis of sea waves is conclusively demonstrated.

S. V. Zhak

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2902. Kokoulin, P. P., The methodology of observations in waves (in Russian), *Meteor. i gidr.* no. 2, 46–47, 1956; *Ref. Zb. Mekb.* no. 12, 1956, Rev. 8305.

A method is described of determining the length of waves by means of a geometric evaluation of ordinary photographs of the waves in question. Author observes that a comparison of the results obtained by this method with the record of an electric wave meter shows satisfactory results.

S. V. Zhak

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2903. Peters, A. S., and Stoker, J. J., The motion of a ship as a floating rigid body in a seaway, *Comm. pure appl. Math.* **10,** 3, 339–490, Aug. 1957.

A linearized theory of the motions of a ship in an infinite, inviscid ocean is developed. Ship is assumed to have the shape of a thin disk, not necessarily flat, in order to create only small disturbances, and theory is based upon the hydrodynamic equations without additional assumptions. Equations are linearized systemmatically by expanding every quantity in a series with respect to a "slenderness" parameter and neglecting squares and higher powers. Theory is applied to three types of ships, one slender about the vertical plane of symmetry (Michell type), a second slender about the water plane (planing type), and a third which is essentially equivalent to the superposition of the first two types (yacht type). The resulting equations, without the necessity of solving them, yield resonant frequencies depending upon the shape of the hull and the mass distribution, and important information about damping of oscillations and interactions between modes of motion.

An explicit solution is given only for a ship form of the Michell type. For the other types the authors suggest that the problem be solved numerically with the aid of high-speed computing equipment, employing various integral equations which are derived. The discussion of the uniqueness of the solutions of these integral equations and the treatment of Green's functions for the various cases are of great mathematical interest in themselves.

Authors are fully aware of the practical limitations of their linearized theory. Reviewer suggests, without detracting from this remarkable and thorough contribution to ship theory, that additional important omissions are neglect of variable added mass and moment of inertia in heaving and pitching and nonlinear damping in rolling.

L. Landweber, USA

2904. Anfimov, V. N., The hydrodynamic characteristics of river craft and the calculation of their roll amplitudes (in Russian), *Trudï Tsentr. n.i-. in-ta rech. flota* no. 3, 55–99, 1955; *Ref. Zb. Mekb.* no. 11, 1956, Rev. 7490.

A method is suggested for approximately calculating the rolling of river craft and inland waterway vessels in waves. The disturbing forces are determined from an approximate hydrodynamic theory of rolling developed by A. M. Bassin from the method of N. E. Zhokovsky, while the hydrodynamics, inertia, and damping forces are evaluated from the results of model experiments. After introducing a number of simplifying assumptions, the practical method of determining the disturbing forces is reduced to calculating the active angle of the wave slope, applying correction terms depending on the influence of the ship's hull, the influence of the ultimate immersion of the ship, and a correction coefficient for the experimental terms introduced. The referred moment of inertia is determined from tank test data on steel crates depending on the structural number. The damping moment is determined on the assumption of its proportionality to the square of the angular velocity of rolling. The proportionality coefficient is determined in relation to the Froude number. Data are furnished on the influence of bilge keels on the rolling characteristics of ship models. The calculation method is illustrated by a numerical example.

Comparison of the results of the theoretical calculation with measured data on the rolling of ship models in waves showed entirely satisfactory agreement between the experimental and the calculated values. Detailed information is given on the tests made by the author with two models of passenger vessels in the naval architecture laboratory of the Leningrad Shipbuilding Institute.

S. N. Blagoveshchenskii

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2905. Nebesnov, V. I., The analytical solution of the equation of motion of a ship's main engine (in Russian), *Nauch. tr. Odessk. in-ta inzh. mor. flota* no. 12, 52–58, 1956; *Ref. Zb. Mekb.* no. 12, 1956, Rev. 8326.

The equation of motion of a ship's main engine working on a screw propeller is examined. Applying a number of simplifications, author concludes that a linear equation, which can be solved by usual methods, sufficiently represents the case.

It is suggested to use this solution to investigate various transitional states of motion of the ship's engine during maneuvers and under way, in particular when going astern.

A. N. Shmyrev

Courtesy Referativnyi Zhurnal, USSR

Translation, courtesy Ministry of Supply, England

2906. Smolyakov, B. N., Calculation of the stability of riveted ship's hulls reinforced with welded elements (in Russian), *Rech. transport* no. 12, 13–19, 1955; *Ref. Zb. Mekb.* no. 12, 1956, Rev. 8638.

Recommendations are made for calculating the general stability of ships with welded and riveted bonds. Descriptions are given of tests to destruction of some special models having a combined welded-riveted seam. Investigations are carried out of the joint

performance of welded and riveted joints at pressures lower than the elastic limit of the rivets. V. S. Chuvikovskii

Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

2907. Kholodilin, A. N., Determination of the coefficient of rolling resistance of a ship under way (in Russian), *Trudĭ Leningr. korablestroit. in-ta* **16**, 157–160, 1955; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8314.

An approximate analysis of the relationship between the coefficient of damping in rolling and the speed of the ship. The underwater part of the hull is regarded as an airfoil of small aspect ratio, in the flow of an ideal fluid. The lift of such an airfoil is calculated. The additional moment of resistance to rolling while under way is determined as the moment of the lifting force with reference to the axis of the rolling ship. An approximate formula is derived, and curves are given, for calculating the relative increment of the coefficient of resistance to rolling of a ship under way.

The results of the calculation according to this formula are compared with experimental data. Yu. M. Guliev

Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

2908. Noguid, L. M., A generalized differential equation for the weights and increase in displacement of a ship (in Russian), *Trudĭ Leningr. korablestroit. in-ta* **16**, 116–122, 1955; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8325.

It is demonstrated that from the starting equation it is possible to derive a generalized, differential equation for the weights, in the following form:

$$dP + \left[\frac{\partial F}{\partial a}\,da + \frac{\partial F}{\partial b}\,db + \ldots + \frac{\partial F}{\partial H}\,dH\right] = \left(\frac{D}{\delta} - \frac{\partial F}{\partial \delta}\right)d\delta + \left(\frac{D}{L} - \frac{\partial F}{\partial L}\right)dL + \left(\frac{D}{B} - \frac{\partial F}{\partial B}\right)dB + \left(\frac{D}{T} - \frac{\partial F}{\partial T}\right)dT$$

in which $F(a, b, c, \ldots H)$ are the weights, depending on L, B, T and the parameters $a, b, c, \ldots H$, where P = sum of independent weights; D = displacement; δ, L, B, T and H, are the coefficient of fullness (block coefficient) and the principal dimensions of the ship; a, b, c, are arbitrary parameters governing the weights, such as the speed of the ship, etc.

If the parameters a, b, c, and H are invariable, the generalized equation becomes transformed into the well-known equation of I. G. Bubnov.

If a given change in weights Δ has to be compensated by a change in one of the parameters δ, L, B, or T, it will be convenient in designing to determine the change in displacement by the formula

$$(d\Delta)_\delta = \Delta_\delta \eta_\delta$$

The coefficients η_δ and the other values,—η_L, η_B, η_T—obtained similarly may be called the coefficients of incremental displacement according to the block coefficient and principal dimensions. L. Ya. Reznitskii

Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

2909. Kholodilin, A. N., The interaction of lateral activated rudders (in Russian), *Trudĭ Leningr. korablestroit. in-ta* **16**, 165–172, 1955; *Ref. Zh. Mekh.* no. 12, 1956, Rev. 8312.

The use of lateral, activated rudders as a means of ship stabilization enables their liveliness to be increased, and the scantlings reduced. Consequently, efforts have been made for many years to apply such arrangements in actual structures. Their introduction is prevented, however, by a number of circumstances, including the loss of efficiency of the rudders owing to interference losses.

The present paper summarizes the results of theoretical and experimental analysis of this problem, conducted by the present author in the Leningrad Testing Tank.

To determine the nature of the interference losses in lateral rudders, a 4-meter model was tested, equipped with two pairs of lateral activated rudders, the interval between which was varied between 13 and 40% of the model length. To eliminate the possibility of errors, such as have been permitted by other research workers, the tests on the traveling model were made in the condition of artificial rolling, induced by the lateral rudders.

The measurements made have shown that the interference losses on the after pair of rudders, working in a turbulent flow, are in fact less than in Allan's tests (Great Britain), made with static deflection of the rudders.

The reasons for this divergence are the presence in front of the forward rudder, as in the case of a swinging wing, of a constant angle of deviation of the flow, reducing the interference losses.

If the distance between the rudders is more than 10 chord lengths, the interference losses may be disregarded.

A. N. Shmyrev

Courtesy Referativnyi Zhurnal, USSR
Translation, courtesy Ministry of Supply, England

Letters to the Editor

2910. Concerning AMR **10**, Rev. 4076 (December 1957): **Donovan, A. F., and Lawrence, H. R., edited by, Aerodynamic components of aircraft at high speeds.**

The above review prompts me to make some observations about the part referring to me.

In the problem of the wing-tailplane interaction, 33 pages have been dedicated to subsonic streams, and 44 pages to supersonic streams. In the problem of wing-body interaction, there are 46 pages on subsonic speeds (of which 17 pages are valid also for supersonic streams), and 86 pages on supersonic speeds. It does not seem to me, therefore, that I have given that prevalence to the consideration of subsonic flows (Review says "incompressible"!!) which Dr. I. Flügge-Lotz complains about. It is true that the study of wing-propeller interaction (47 pages) has been made only for subsonic streams, but it is also true that the consideration of the analogous problem in supersonic streams would have required, and would still require an ex-novo study. Neither has it been possible to consider the problem of drag in the wing-body interaction, because at the time the manuscript was delivered (1955) studies on the subjects had only just been declassified.

The program of Section C of the book had been established in agreement between the Scientific Committee and the author as early as 1950. I have done nothing but develop the problems that were established in this program. There has, therefore, been no inclination on my part to overcome the wishes of the Editor (without considering the fact that my scientific activity has been essentially directed to supersonic aerodynamics). Dr. Flügge-Lotz says: "Ferrari presents the widely scattered material on interaction in great detail. This saves looking up the cited papers". This, in my opinion, is especially useful since the reader, in many of the problems considered by me, would not be able to find in the "cited papers" the developments discussed in the chapters written by me. With reference to these, may I say that I have tried to give an understandable explanation not only of the result ob-

tained, but also of the methods followed; this has been for me a very considerable task because of the complexity of the analytical developments that were often necessary.

C. Ferrari, Italy

2911. Concerning AMR **11**, Review 8 (January 1958): **Yu, Y.-Y., On the generalized ber, bei, ker, and kei functions with application to plate problems:**

Author does not agree with the reviewer of the above paper. Firstly, the functions under consideration *are not* the Bessel functions $J_n(x)$ and $Y_n(x)$ but *are* the real and imaginary parts of the latter functions. Secondly, the discussion on these functions is just like any other discussion on the similar Ber, Bei, Ker, Kei, Her, Hei, Yer, Yei, Ster, and Stei functions. Discussions on the latter group of functions and their applications would all become trivial on the basis of the reviewer's argument, since all of them are nothing else but the real and imaginary parts of some Bessel functions with complex arguments.

Y.-Y. Yu, USA

2912. Concerning AMR **11**, Rev. 314 (January 1958): **Troshin, Ya. K., Generalization of . . . . . . equations for nonsteady process of flame propagation in pipes.**

In the above review there is a fairly obvious error which probably arose in transliterating from Russian. It is clearly "Hugoniot's equations and Hugoniot's adiabatics" which are referred to.

W. Squire, USA

2913. Regarding AMR **11**, Review 1042 (March 1958): **Klein, G., A contribution to flame theory,** *Phil. Trans. roy. Soc. Lond.* (A) **249**, 967, 389-415, Feb. 1957.

The above paper was reviewed by C. Franz instead of W. Jost. The editors regret this error.

Books Received for Review

ALBLAS, J. B., Theorie van de driedimensionale spanningstoestand in een doorboorde plaat, Amsterdam, H. J. Paris, 1957, xii + 127 pp.

BECKERS, H. L., Heat transfer in turbulent tube flow, Gravenhage, Martinus Nijhoff, 1956, 52 pp. (Paperbound)

BETHE, H. A., Splitting of terms in crystals, New York, Consultants Bureau, 1958, 69 pp. $3.00. (Paperbound)

BOLZ, R. W., ASME handbook—metals engineering processes, New York, McGraw-Hill, 1958, xv + 428 pp. $13.50.

BURTON, R. A., Vibration and impact, Reading, Massachusetts, Addison-Wesley Publishing Co., 1958, x + 310 pp. $8.50.

CHEN, K.-K., Summation of the Fourier series of orthogonal functions, Peking, China, Science Press, 1957, iv + 174 pp.

CHILLON, P., Resistance des Materiaux, Tome I, Biblioteque de l'enseignement technique, Paris, Dunod, 1957, viii + 317 pp.

EIRICH, F. R., edited by, Rheology; theory and applications, Vol. II, New York, Academic Press, Inc., 1958, xiii + 591 pp. $18.00.

GROEN, L. J., Kinetiek van de allotrope omzetting in tin, Delft, Technische Wetenschap ann de Technische Hogeschool, 1956, 117 pp. (Paperbound).

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